JENSEN

The Economical Design of a Reinforced Concrete Building

Civil Engineering

C. E.

1914

THE ECONOMICAL DESIGN OF A REINFORCED CONCRETE BUILDING

BY

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B. S. UNIVERSITY OF ILLINOIS 1906

THESIS

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF

CIVIL ENGINEER

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1914

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 6, 1914

I HEREBY RECOMMEND THAT THE THESIS PREPARED BY

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ENTITLED The Economical Design of a Reinforced Concrete Building

BE ACCEPTED AS FULFILLING THIS PART ON THE REQUIREMENTS FOR THE PROFESSIONAL DEGREE OF Civil Engineer

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Committee

PREFACE.

This thesis is the outcome of the writer's experience in reinforced concrete construction. As designing engineer in the offices of consulting engineers, he became familiar with current engineering practice. In his capacity of field engineer for these same men, he learned some features which are sometimes overlooked in the office. His experience as contracting engineer forcibly brought home to him the economics side of the question. This side was always especially interesting to him. It always was in his mind, as in his official capacity, the building work of a great city was presented to him for consideration and approval. All the experience in building construction which he has had, contributed to this thesis.

In the issues of the Engineering News of Aug.3,1911, and June 6,1912, two chapters of this thesis were published, and subsequently discussed in the columns of the same paper.

Each chapter is considered as a subject complete in itself, and conclusions reached independent of the other chapters. In the five chapters all the economics of a panel extending from the roof to the foundations were discussed. It is hoped that the conclusions at the end of each chapter will be of general interest.

March 10,1914. J.Norman Jensen.

CONTENTS.

CHAPTER I.

The Economics of the Reinforced Concrete Floor Slab.

What are the most economical unit stresses in both concrete and steel in the design of any reinforced concrete floor slab? This is the question that arises in the mind of the engineer who wishes to design so that the money expended in any particular building is wisely spent. In various treatises on reinforced concrete there are elaborate equations and curves showing, for instance, that low unit stresses represent maximum economy in slab design. ~~He~~ The engineer knows that these low unit stresses will thicken up his slab, increase the dead load on his beams and girders, make the columns, already too large on the lower floors, larger than ever, and finally run up the cost of his footings. He is inclined to doubt the statements given in these treatises.

To the experienced designer one fact is forcibly brought home to him, and that is that the dead load represents too large a proportion of the total load in the average reinforced concrete design. How can he reduce this dead load? There are two ways open. One is the use of a larger percentage of steel, and the other is the use of a richer mix in the slab. Both ways ensure a thinner slab, and therefore a smaller dead load. It is the intention of this chapter to discuss whether these methods are economical or not.

The writer does not believe in applying the methods of the differential calculus in arriving at conclusions

as to what constitutes maximum economy in the subject under consideration. He believes that when the owner's money is involved it is not wise to lean too heavily on a differential equation. To him the rational method is to make a great many designs, and compare them.

DESIGNING DATA.

In making comparative designs it is advisable to use stresses that are sanctioned by good authority. In the Middle West the Chicago Building Ordinance is generally accepted as the standard. This ordinance provides that in the design of slabs and beams:

(a) The common theory of flexure should apply.

(b) The steel should take all the direct tensile stresses.

(c) The stress strain curve of concrete in compression is a straight line.

Although the most common mix for slabs is the 1:2:4 mix, provision is made in the ordinance for richer mixes. The allowable unit stresses and the ratio of the moduli of elasticity of steel to the various mixes is given below.

Mix	Unit Stress lbs. per sq. in.	Ratio of the Moduli of Elasticity.
1:2:4	700	15
1:1-1/2:3	840	12
1:1:2.	1015	10

The allowable unit stress for high elastic steel is 18000 lbs.per sq.in.

In the design of these slabs the notation and formulas found in standard text books on reinforced concrete have been used.

NOTATION AND FORMULAS.

f_c = allowable unit stress in concrete.

f_s = ,, ,, ,, ,, steel .

n = ratio of the moduli of elasticity of concrete to steel

M = bending moment or resisting moment.

A = steel area per foot of width of slab.

b = width of slab.

d = depth of slab to center of steel.

h = total thickness of slab.

K = constant = coefficient of strength.

p = A/ bd.

$$K = \frac{f_c^2 \, n \, (3 f_s + 2 n f_c)}{6 (f_s + n f_c)^2}$$

$$p = 1 / 2 \, \frac{1}{\frac{f_s}{f_c} \left(\frac{f_s}{n f_c} + 1 \right)}$$

$$d = \sqrt{\frac{M}{Kb}} , \qquad A = p \, b \, d.$$

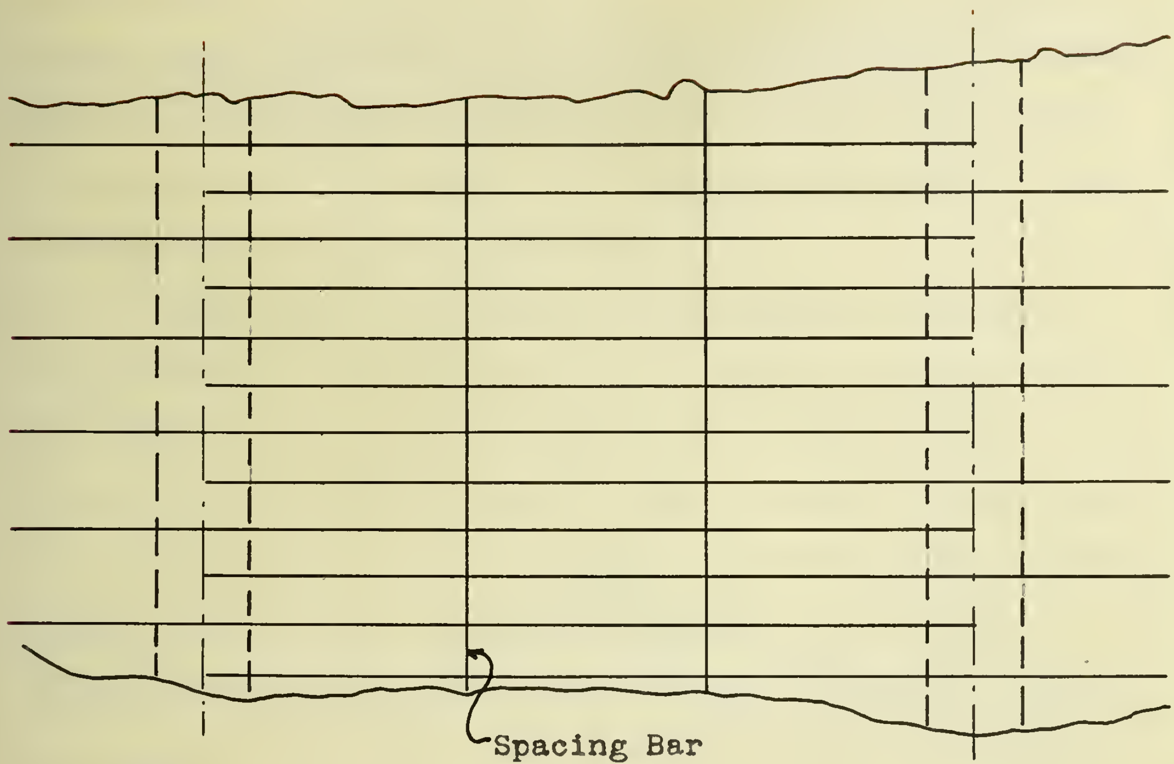
Values for (p) and (K) for different values of (n), (f_c), and (f_s) are given in Table I. These working stresses are arranged in ascending order of (K). For any one mix the values of (p) are also arranged in the same order. More values are given for the 1:2:4 mix than for any others, as it is the almost universal mix for building

construction. The letters used in the first column in Table 1 are given as a convenient way of designating the unit stresses for the given mix.

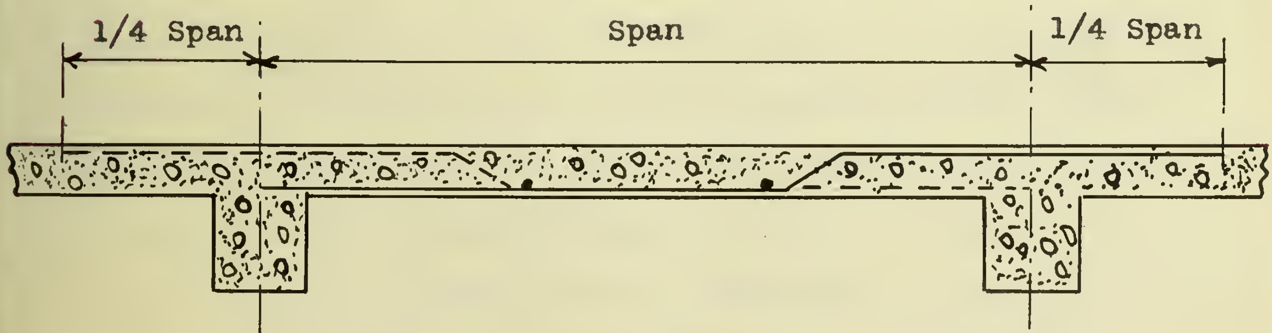
In making an analysis of costs, slabs reinforced in one direction were designed for superimposed loads of 50, 75 , 125 , and 225 lbs. per sq. ft., for spans varying by 2'-0" from 8'-0" to 16'-0" inclusive. Slabs reinforced in two directions were designed for superimposed loads of 75, 125 , and 225 lbs. per sq.ft., for spans varying from 14'-0" to 22'-0" inclusive. In both cases for the same span and the same load, different mixes and percentages of steel were used. Each span and each load was analyzed using each of the combination of stresses given in Table 1.

TABLE 1.

No.	Mix	n	f_c	f_s	p	K
A1	1:2:4	15	600	18000	.0056	89
A2	,,	,,	650	18000	.0064	101
A3	,,	,,	700	18000	.0072	113
A4	,,	,,	,,	16000	.0087	120
A5	,,	,,	,,	14000	.0107	129
B1	1:1-1/2:3	12	840	18000	.0084	133
B2	,,	,,	,,	16000	.0101	142
B3	,,	,,	,,	14000	.0126	152
C1	1:1:2	10	1015	18000	.0102	161
C2	1:1:2	,,	,,	16000	.0124	171
C3	,,	,,	,,	14000	.0152	184



PLAN.



SECTION

Fig.1.

By superimposed load is meant the live load plus the weight of the flooring, etc., on top of the concrete slab. Thus a superimposed load of 75 lbs. per sq. ft. is a live load of 50 lbs. required by the ordinance for office buildings plus the weight of a wooden floor laid on sleepers embedded in cinder concrete.

A plan and section showing the typical arrangement of slab bars is given in Fig. 1. It will be noticed that each bar extends from the middle of the beam to the quarter point of the adjoining span. In this way the same steel is provided over the support as in the middle of the span. This was done as the slab under consideration is supposed to be a typical interior panel in which a bending moment of 1 / 12th over the support and the middle of the span is required by the ordinance. In slabs reinforced in two directions, the same general arrangement of steel is employed, and the same bending moments is used; but only one-half of the total load is taken in each direction. The span is taken from center to center of supports.

METHOD OF COST ANALYSIS.

In the small space allotted for the discussion of this chapter, it is manifestly impossible to reprint all the tables, etc., upon which the final conclusions were based. Table 2 gives the results of slabs designed for a superimposed load of 125 lbs., using a 1:1-1/2:3 mix for the spans given. The letter used on the left side of the table refers to the same letter given in designating the constants in Table 1. The other nomentclature used has

already been defined. In designing any particular slab, a total thickness (h) was assumed, and the dead load of such a thickness was added to the superimposed load in obtaining the bending moment (M). The depth of slab to center of steel (d) was then calculated using the (K) obtained from Table 1. If on adding 3/4" to the lighter slabs and 1" to the heavier ones to the theoretical depth found, the total thickness (h) was the same as the assumed thickness, the bending moment values were accepted, otherwise the computations were gone over again until the assumed and final thicknesses agreed. These thicknesses of slab were given to the nearest 1/4 inch. While this may seem a needless refinement, it was necessary to work that close in order to arrive at correct conclusions.

Knowing (d), the steel area per foot of width of slab (A) was obtained using the corresponding (p) in Table 1. The weight of steel per square foot was obtained by multiplying (A) by 3.4, and then adding 25% additional due to negative bending moment. (see Fig.1). Shrinkage steel or spacing bars were not included in these weights. The concrete quantities are given in cubic feet, and the total cost in cents per square foot.

In looking over this table it will be seen that the bending moment varies for the same span. This is because of the fact that in the slab the dead load is such a large proportion of the total load that any slight increase in the total thickness will materially effect the bending moment.

UNIT COSTS.

In the following table the cost of steel in place has been estimated at 3¢ per lb. The cost of concrete per cubic foot has been taken as follows:

A	1:2.4	21¢
B	1:1-1/2:3	23¢
C	1:1:2.	26¢

It is believed that these costs are representative of the unit costs on any large job in the Middle West. It is realized that it is not wise to rely wholly on these unit costs, and cost analyses have been made on the basis of steel at 2-3/4¢ per lb., and concrete 2¢ higher per cubic foot than given above. In no case has the cost of the formwork been included in the total cost as it would be practically the same for all slabs.

The total costs for each span and superimposed load were tabulated. These costs were based on the values given above. Using the same quantities of steel and concrete, new costs were figured on the basis of unit costs of concrete as given above, and unit steel at 2-3/4¢ and 3¢ per lb., and also unit costs of concrete 2¢ higher than those given above, and steel at 2-3/4¢ and 3¢. In this way four different combinations of unit costs were obtained. It is thought that these four combinations will cover the ordinary variations in unit costs.

TABLE 2.

Superimposed Load = 125#, 1:1-1/2:3 Concrete.

		One-way Slab.			Two-way Slab			
	Span	12'0"	14'0"	16'0"	16'0"	18'0"	20'0"	
	M	2260	3220	4400	2010	2620	3330	ft,lbs.
	d	4.12	4.91	5.75	3.88	4.44	5.00	ins,
	h	5	5-3/4	6-1/2	5	5_1/2	6	ins.
B1	A	.41	.49	.58	.39	.45	.51	sq.ins.
	Steel	1.74	2.08	2.47	3.31	3.82	4.33	lbs,
	Concrete	.42	.48	.54	.42	.46	.50	cu.ft-
	Total Cost	14.8	17.2	20.8	19.5	22.0	24.5	¢
	M	2220	3170	4320	1970	2580	3280	ft.lbs.
	d	3.94	4.72	532	3.73	4.26	4.80	ins.
	h	4_3/4	5_1/2	6-1/4	4-3/4	5-1/4	5-3/4	ins.
B2	A	.48	.57	.67	. 45	.52	.58	sq.ins.
	Steel	2.04	2.42	2.85	3.82	4.41	4.92	lbs.
	Concrete	.40	.46	.52	.40	.44	.48	cu,ft.
	Total Cost	15.2	17.8	21.6	20.6	23.2	25.8	¢
	M	2170	3120	4320	1970	2580	3280	ft.lbs.
	d	3.78	4.52	5.32	3.60	4.11	4.64	ins.
	h	4-1/2	5-1/4	6-1/4	4-3/4	5-1/4	5-3/4	ins.
B3	A	.57	.68	.81	.54	.62	.70	sq,ins.
	Steel	2.42	2.89	3.44	4.59	5.26	5.95	lbs.
	Concrete	.38	.44	.52	.40	.44	.48	cu.ft.
	Total Cost	15.9	18.7	22.3	22.9	25.8	28.9	¢

DISCUSSION.

On comparing the four different combinations of unit costs it was discovered that the same combinations of unit stresses for any particular load and span always gave the same relative result. If certain stresses were most economical for any one of the four combinations, they were also most economical for all of them.

In arriving at a decision as to the economy of different designs, it was necessary to make allowance for the decrease in thickness for the richer mixes and the higher percentages of steel. An inspection of Table 2 brings out the fact that higher percentages of steel mean a thinner slab. A thinner slab would of course mean less dead load on the beams, girders, columns; and this would mean that less steel and concrete would be required in the panel to carry the superimposed load. It is hard to give an exact figure as to how much a saving in cost would be represented by a less dead load, but an estimate of a typical panel seemed to indicate that the decrease of one inch in the thickness of the slab would mean the decrease of one cent per square foot in the cost of the panel.

CONCLUSIONS.

On the above basis designs were compared. For any "mix" the most economical design for any span and any load was that in which the maximum unit stresses in both the concrete and the steel were used. That is to say, the unit stresses of 700 and 18000 (A3) were the most econom-

ical for the 1:2:4 mix, 840 and 18000 (B1) for the 1:1-1/2:3 mix, and 1015 and 18000(C1) for the 1:1:2 mix. The evidence that low unit stresses in either concrete were uneconomical was quite conclusive. These facts were true in both the one-way and the two-way slabs.

Comparing the one-way and the two-way slabs for the same mix, span, and load, it was found that the two-way slabs were decidedly more economical as they were both lower in cost, and the slabs were considerably thinner. The economy was more apparent than real as the cost of the additional beam required in a panel reinforced in two directions offsets the economy in the slab, and the panel taken as a whole is more costly than the panel reinforced in one direction only.

The series was computed with especial reference to the economy of the rich mix in a slab, if such could be proven. There was not much variation in the cost of two slabs of the same span and load, but of different mixes. Keeping in mind the assumption that a saving of one inch in the thickness of the slab would mean a saving of one cent per square foot in the cost of the slab, it was found, in general, that the 1:1-1/2:3 mix was more economical than the 1:2:4 mix for the one-way slabs, and that the 1:1:2 mix would not be considered economical at all. In the slabs reinforced in two directions, the 1:2:4 mix was found to be the most economical almost without exception.

The Economics of Reinforced Concrete Beams.

A rule of thumb frequently used in the design of reinforced concrete beams is as follows:

> Make the depth in inches to the center of the steel the same as the span, center to center of supports, in feet.

It is the intention of this article to arrive at some conclusion as to the most economical depth of reinforced concrete beams ordinarily used in building construction.

It would seem that the deeper the beam the more economical it is, because of the decrease in the steel used. The increased depth, however, means increased cost of formwork with about the same amount of concrete. In order to find the true cost of any beam it will therefore be necessary to find the total cost of the three items, of concrete, steel, and formwork.

This last item of formwork has driven many a contractor to the rocks. It is such an important item that in most jobs it amounts to at least one-quarter of the total cost of the reinforced-concrete contract. It would seem, therefore, that any formula which does not include this item in the economical depth of a concrete beam would be greatly in error.

It is not the writer's ambition to derive a new formula giving the conditions for maximum economy. Taking into account the three variables in cost given above, such a formula would become so cumbersome as to be prac-

tically useless. It is the writer's wish to arrive at some conclusion which in his opinion would be a safe guide for to him and to others as to this subject of economical design.

In order to arrive at a conclusion it was necessary to design a great many beams. A total load of 25,000 lbs. uniformly distributed was assumed, and beams were designed for spans of from 14 to 18 feet. For each span a beam equal in depth in inches to the span in feet was designed, and then for the same span and load four other beams with depths in inches 2 in. and 4 in. less and greater than the span in feet were also designed. In the same way a total load of 50,000 lbs. and spans of from 14 to 22 feet, and a total load of 75,000 lbs. and spans of from 20 to 24 feet were assumed. In this way loads, spans, and beams were obtained covering all the cases ordinarily occurring in building design. For practical reasons minimum widths of 6 in. and maximum widths of 16 in. were adopted for the beams.

BUILDING ORDINANCES.

All designs were based on the Revised Building Ordinances of the City of Chicago. The unit tensile stress in the steel was taken at 18,000#, and the bond stress at 100# per sq.in. The vertical shear measured on a section of the beam between the centers of action of horizontal forces was assumed at 133# per sq.in. The

bending moment in the middle of the span and at the support was taken as $1 / 12\ wl^2$. The span used was the distance between centers of supports. The steel in the beam was designed so that it was nowhere nearer the surface than 1-1/2 ins., and the thickness of concrete between the separate pieces of steel was not less than 1-1/2 times the diameter of the steel.

NOTATION AND FORMULAS.

f_s = unit tensile stress in steel.

M = bending moment

A_s = steel area

d = depth of beam to center of steel

b' = width of web of beam

V = total shear

v = shearing unit stress

u = bond stress per unit area

o = perimeter of bar

Σ_o = sum of perimeters of all bars

h = total depth of beam.

$$A_s = \frac{M}{7/8\ f_s d} \qquad (1)$$

$$v = \frac{V}{7/8\ b' d} \qquad (2)$$

$$u = \frac{V}{7/8\ d\ \Sigma_o} \qquad (3)$$

A section and elevation of a typical beam is given in Fig.1. It will be seen that half of the bars in the bottom are bent up and extend to the quarter point of the

span beyond. In this way some reinforcement is provided for diagonal tension, and also the same amount of steel is provided over the support as in the middle of the span. The number and spacing of the stirrups is typical as these would vary with the load on the beam.

It was thought advisable to give only a small part of the mass of data computed . Table I gives the designing data for a beam carrying a total uniform load of 50,000 lbs. with a span of 22 ft.

TABLE I.

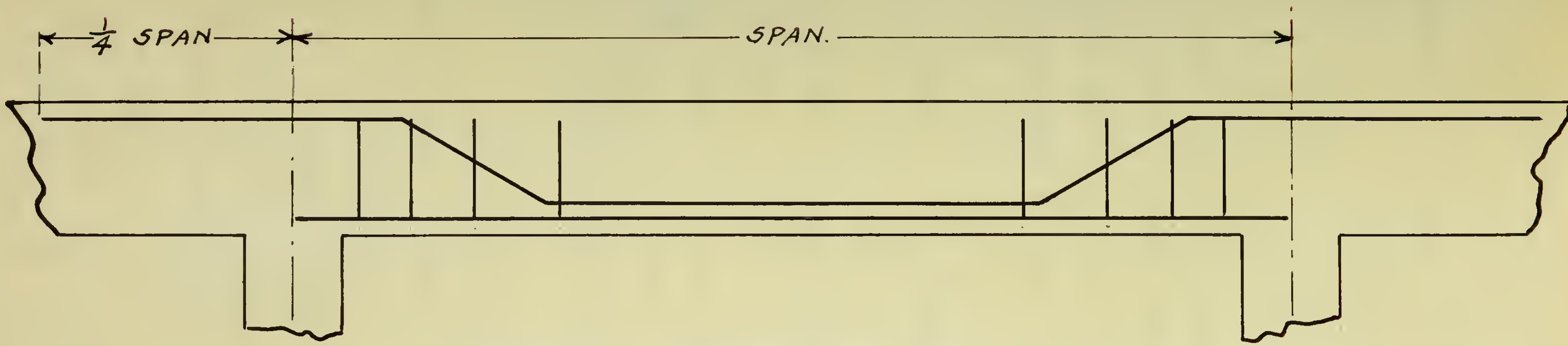
Total load 50,000#   Span = 22 ft.

	d	b'	M,	A_s	sq.	u	h
No.	ins.	ins.	ins.lbs.	sq.ins	bars	lbs.	ins.
36	18	12	1,100,000	3.88	5-7/8	91	21
					5-3/4		
37	20	11	,,	3.50	1-7/8	78	23
38	22	10	,,	3.17	4-7/8	93	25
39	24	9	,,	2.92	5-3/4	80	27
					2-3/4		
40	26	8	,,	2.68	2-7/8	85	29

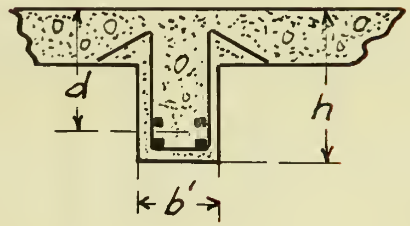
In order to show how these data were obtained, the complete computations for one of the beams will be carried through. Let us take beam No. 38. With a total load of 50,000# and a span of 22 ft., $M = 1/12 \times 50000 \times 22 \times 12 =$ 1,100,000 in.lbs. From formula (1), $A_s = \frac{1,100,000}{7/8 \times 18000 \times 22} =$ 3.17 sq.ins. From formula (2) $v = \frac{25000}{7/8 \times 10 \times 22} = 130\#$

4-7/8" square bars were assumed, $A_s = 3.04$ sq.ins. From formula (3), $u = \frac{25000}{7/8 \times 22 \times 4 \times 3.5} = 93\#$. h = 22+3=25".

In the analysis given above the beam is assumed to



ELEVATION



SECTION

FIG. I.

be a T section with sufficient width of flange so that the compression in the top is within the limits required by Ordinance. Formula (1) is approximate but sufficiently accurate for the purpose since the width and thickness of the flange are not known. Formula (2) is used to assure a width corresponding to the unit shear on a beam fully reinforced with inclined bars and stirrups. Formula (3) is used to ensure that too large bars are not used, and that the values for bond are not exceeded. In many cases a beam with a fixed depth must be widened because of the limitations of bond stress and spacing of bars in a beam, as given in the Ordinances quoted above.

The computations as to the size and spacing of the stirrups are not given. This is because of the fact that for the same total load and the varying spans and depths, the amount of steel in stirrups, and therefore their cost, would be about the same. In no case is the cost of these stirrups included in the cost of the steel in the beams.

COSTS.

The beams under discussion are supposed to be typical, that is, there are supposed to be a great many of the same kind in a large job. The cost in place of the 1:2:4 concrete assumed has been estimated at 20¢ per cubic foot, the cost of the steel in place has been estimated at 3¢ per lb. The formwork has been taken at 10¢ per square foot.

QUANTITIES.

The quantities for the beams given in Table I are given in Table II.

TABLE II.

Quantities & Costs per Lineal Foot.

No.	Concrete Cu.Ft.	Steel Lbs.	Forms Sq.Ft.	Concrete cents	Steel cents	Forms cents	Total cents
36	1.75	16.5	4.5	35.0	49.5	45.0	129.5
37	1.76	14.9	4.8	35.2	44.7	47.5	127.4
38	1.74	13.5	5.0	34.8	40.5	50.0	125.3
39	1.69	12.4	5.3	33.8	37.2	52.5	123.5
40	1.61	11.4	5.5	32.2	34.2	55.0	121.4

The method of obtaining these quantities will be given for beam No.38. Since the width (b') and the total depth (h) are 10" and 25", the concrete per lineal foot is $\frac{10x25}{144}$ = 1.74 cu.ft. The formwork is $\frac{10+2x25}{12}$ = 5 sq. ft. Since the steel is lapped at the support, the weight of steel per lineal foot of beam is 3.17x1.25x3.4 = 13.5 lbs.Using the unit prices given above, the rest of the table is self explanatory.

CONCLUSION.

Table II brings out the facts which are representative of the whole series of beams. The beam which has the same depth in inches as the span in feet, is more economical than any of the shallower beams of the same load and span, but [illegible] it is less economical than the deeper beams. In the lighter loads the difference in cost is so slight that the rule of thumb given above is

correct enough for all practical purposes. In the heavier loads there is no doubt that the deeper the beam the more economical it is, as the increased cost of formwork is not enough to compensate for the saving in the cost of the steel. Whether such increased depths would interfere with the architectural features of the building or not, is a matter which must be decided in each case. But as far as the beams themselves are concerned, the above conclusions are true.

CHAPTER III

The Economical Design of a Reinforced Concrete Floor Panel.

Because of the fact that there are at least three variables in the cost of any reinforced concrete work, an economical design of a reinforced concrete floor is oftentimes difficult of solution. It has been the aim of some writers to express the relations necessary for maximum economy by means of a formula. The aid of the calculus has even been invoked in its derivation. In every case the cost of the formwork has not been included in the formula given, or has been lumped in "one glorious integer" with the cost of the concrete. When it is remembered that the cost of formwork may be one third of the cost of the job, it is readily seen that this is too big an item to be overlooked.

The curling tail of an integral sign may express results which are of more or less value, but to the writer it seems that the "cut and try" method is the best ~~in~~ for the problem at hand. Instead of taking the slab or the beam as a measure of the relative economy of design, a whole panel has been used, as it is believed that this is the logicalway to study any design. When the panel loads are known the design of columns and footings may be worked out to the best advantage.

A typical interior panel has been designed in fourteen different ways, and an analysis made of the cost of each. The whole design was based in general on the Revised Building Ordinance of the City of Chicago. A summary of the

sections which have a direct bearing on the design of the panel is given verbatim below. In order that all designs should be on the same basis it was necessary to impose in each case the same general conditions of design.

GENERAL CONDITIONS OF DESIGN.

Size of panel, 20 x 20 ft.

Live load = 100 lbs per sq. ft.

Dead load = Weight of 1-in. granitoid floor finsh + plaster ceiling + floor construction.

Ceiling to be plastered. Flat ceiling preferred.

Maximum depth of beam or girder allowed = 24 ins.

1: 2: 4. Concrete.

Summary of Building Ordinances Bearing on Design.

Unit tensile stress in steel	= 18,000	lbs.	per sq.in.
" Bending compression in extreme fiber	= 700	"	"
" tension in concrete on diagonal plane	= 40	"	"
" bond stress	= 100	"	"
Ratio of moduli of elasticity	= 15.		

The steel to take all the tension.

The stress-strain curve of concrete in compression is a straight line.

Bending moment in middle of intermediate span= $\frac{wl^2}{12}$.

Span for freely supported slab = the free opening plus the depth.

Span for continuous beams = distance between centers of supports.

Web reinforcement provided where vertical shear measured on the section of a beam or girder between the centers of action of the horizontal stresses exceeds 40 lbs per sq.in. Vertical shear measured as stated above shall in no case exceed 133 lbs. per sq.in.

For T-beams the width of the stem only shall be used in calculating the above shear.

Width of flange in T-beams limited to a width of 1/3 the span of rib, and also to 3/4 the distance c. to c. of ribs.

Where reinforced-concrete girders support reinforced beams, the portion of floor slab acting as a flange to the girder must be reinforced with rods near the top, at right angles to the girder.

Steel shall nowhere be nearer the surface of the concrete than 1-1/2 " for beams and girders, and 1/2" but not less than the diameter of the bar for slabs.

Thickness of concrete between the separate pieces of steel in beams and girders shall not be less than 1-1/2 times the maximum sectional dimension of the steel.

For square slabs with two-way reinforcements the bending moment at the center of the slab shall not be less than $\frac{wl^2}{24}$ for intermediate span.

The formulas used in the design are those recommended by the Joint Committee on Concrete and Reinforced Concrete. In some cases transposition of some of these formulas has been used, and additional formulas derived in the same general way have been utilized, The standard notation adopted is as follows:

M = bending moment or resisting moment.

A = steel area.

b = width of beam.

d = depth of beam to center of steel.

k = ratio of depth of neutral axis to depth d.

j = ratio of lever arm of resisting couple to depth d.

K = constant = coefficient of strength.

$p = \dfrac{A}{bd}$.

z = depth of resultant compression below top.

jd= d-z = arm of resisting couple.

b = width of flange of T beams.

b'= " " stem " " .

t = thickness of flange of T beam.

V = total shear.

v = shearing unit stress.

u = bond stress per unit area of bond.

Vc= total allowable shear on bd.

s = spacing of stirrups.

o = perimeter of bar.

Σo = sum of perimeters of all bars.

The following formulas have been used in the design.

Rectangular beams and slab.

$$K = \frac{f_c^2 n (3 f_s + 2n f_c)}{6 (f_s + n f_c)^2}$$

$$p = \frac{1}{2} \frac{1}{\frac{f_s}{f_c}\left(\frac{f_s}{n f_c} + 1\right)}$$

$M = Kbd^2$, $A = pbd$.

$k = \sqrt{2pn + (pn)^2} - pn$ $j = 1 - \frac{1}{3}k$.

$$Kd = \frac{n f_c d}{f_s + n f_c}$$

$$Z = \frac{3Kd - 2t}{2Kd - t} \; \frac{t}{3}$$

$$A = \frac{M}{f_s\,(d - z)}$$

$$b = \frac{2n\,(d - Kd)}{t\,(2Kd - t)}\; A$$

Bond and Shear.

$$U = \frac{V}{7/8\, d\, \Sigma_o}$$

$$v = \frac{V}{7/8\, b'd}$$

$$s = \frac{7/8\, d\, f_s A}{V - V_c}$$, A being area of section of stirrup

The three variables entering into the cost of a reinforced concrete job are steel, concrete, and form-work. In some of the designs the cost of hollow tile enters in as a fourth variable.

Steel is usually quoted as base, f. O. B. Pittsburg. By the term base is meant the price per 100 lbs. for 3/4" bars or larger. There are additional charges for bars smaller than 3/4", 5/8" bars costing 5¢, 1/2" bars 10¢, 3/8" bars 25¢, and 1/4" bars 50¢ per 100lbs. more than the base price. That is to say that if the price f. O. b. Pittsburg is $1.35 base, in car load lots,

the price for 1/2" bars would be $1.45 per 100 lbs. To these prices f. o. b. Pittsburg must be added the freight rate to the job in question. For instance the car load rate on bars to Chicago from Pittsburg is 18¢ per 100 lbs. The base price fluctuates more or less, but at present writing is $1.35 f. o. b. Pittsburg for deformed bars, cut to lengths.

The unit prices for the steel given for each design, are the prices for the steel in place. These include the cost of hauling to the job from the cars, unloading, bending slab bars, beam bars, and stirrups, placing into the forms, and doing all necessary wiring.

Concrete may be subdivided into the cost of the cement, sand, gravel or crushed stone, water and the labor in mixing and placing. As a general rule the cement mill nearest the job can furnish the cement the cheapest, as freight charges are a big item in the cost.
The length of haul from the cars to the job is another large factor in the cost of the cement. Sand and gravel or crushed stone are usually quoted f. o. b. the job, the distance from the job to the local pit or quarry entering largely in the cost of the material. Water is generally a small item and does not run more than 3¢ a cubic yard. Cement in the Chicago market is quoted at $1.25 per barrel, sand $1.00 per cubic yard, gravel $1.45 per cubic yard, crushed stone $1.65 per cubic yard. The above prices are all f. o. b. the job for an ordinary haul. The labor of mixing by machinery, hoisting, and

wheeling has been placed at $1.70 per cubic yard. This last item also includes a small allowance for the incidental expenses incurred by the hoisting and mixing plant, and also an allowance for the unavoidable delays which always occur on any job.

In the cost of formwork given for each design it is assumed that the floor framing is typical and that the forms can be used three times. The labor cost includes the cost of making, setting, bracing, removing. cleaning, and erecting again. The cost of the material includes nails, wire, and all lumber used in the forms, also the posts and braces used in supporting the same. The cost of lumber has been figured at $22.00 per M for yellow pine. The cost of labor varies with the design, and is included in the unit cost given for each case.The cost of slab and beam forms is given as so many cents per square foot. Slab forms are measured center to center of columns. Beam forms are measured by the square feet of surface in contact with the concrete. Thus a beam 12" x 24" under the slab would have five square feet of forms per lineal foot of beam. Beams are measured face to face of girders, and girders face to face of columns in computing the length of each. It is assumed that the girders frame into an 18" column in each design.

The present union scale for form carpenters is 60¢per hour, and for laborers 37 1/2¢ per hour for an eight hour day. The unit prices given in this article do not include contractor's profits.

A plan and a section of a typical interior panel is shown in each design. The plan is taken looking upwards. The designs in which hollow tile is used show the concrete ribs between the tile. Hollow tile floors would present a flat ceiling and there would be no line of demarcation between the hollow tile and the concrete rib; but it is believed that the design is more clearly brought out by showing the floor this way. In some of the last designs the flare of the column and the square column cap ~~is~~ are indicated. In each case a half section is given of double the scale of the plan.

Special items of interest in the design, construction, and cost of each design will be taken up in turn. In no case is the steel shown, but a general description of the type adopted is given in each.

DESIGN NO. 1.

In this design is shown the common form of construction in which the load is carried from the slab into the beam, from the beam into the girder, and thence into the column. The full live load has been taken on the girders. This and the next design represent the earlier forms of framing in which the beams and girders have been laid out in imitation of standard steel framing.

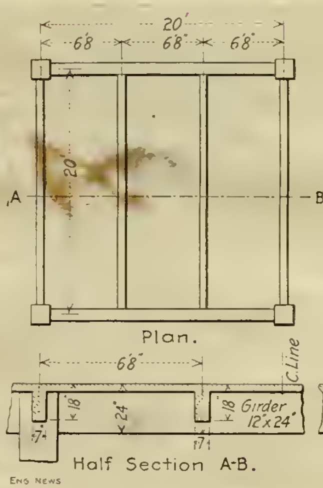
The slab has been designed as a continuous beam, with the same steel over the support as in the center of the span. This is accomplished by bending each slab bar at one end in order to provide for negative bending moment. The straight end of each bar bears on the beam and the

bent end terminates at the quarter point beyond the center line of the next beam. The bent and the straight ends of the bars alternate. This method of placing slab bars saves considerable steel, and it is believed gives more satisfactory results than using straight top bars. The trouble with straight slab bars over the beams is that one never knows whether or not those top bars are in the position they are intended to be ~~or not~~, as there is always danger that they may sink into the "soupy" concrete. Slab bars as described above can not become misplaced if they are properly bentand placed in the bottom of the slab. It is believed that the increased expense of bending these slab bars is off-set by the saving in the steel,and a better type of construction results.

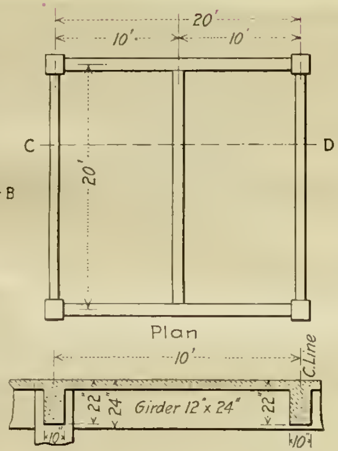
A lower unit stress in the concrete than allowed in the Ordinance has been used because of the small bending moment. In order to provide for temperature stresses and to tie the main reinforcing bars together,3/8" square bars 24" C.to C. have been used.

The beam has been designed as a T beam. Half of the bars in the bottom of the beam have been bent up at the quarter point,and extend to the quarter point beyond the center line of the column in both directions. In this way the same amount of steel is obtained over the support as in the center of the span. Ordinary 3/8" round single loop "U" stirrups have been used.

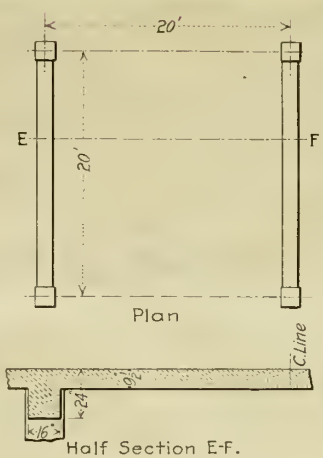
In the design of the girder the bending moment has



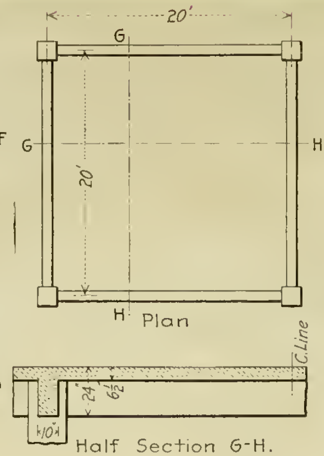
Design No. 1.—Solid Slab, Intermediate Beams at Third Points.



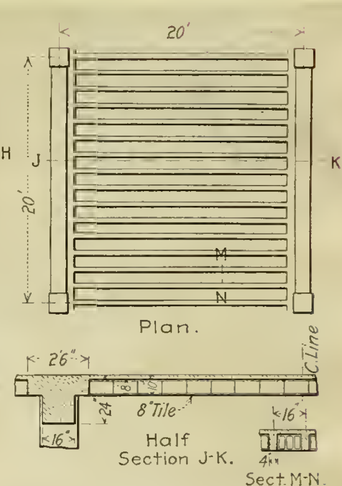
Design No. 2.—Solid Slab, Intermediate Beam in Middle.



Design No. 3.—One Way Solid Slab.



Design No. 4.—Two Way Solid Slab.



Design No. 5.—One Way Hollow Tile Slab.

FIGS. 1-5.

been figured as a simple supported span, and then a reduction has been made for continuity. Bars have been bent up similarly to the beam, and a close spacing of stirrups maintained to take the diagonal tension. In order to provide for loads which might be transmitted directly to the girder, 1/2" square top bars 5 ft. long were placed near the top of the slab over the girder for its full length.

The cost of the steel has been placed at 3¢ per lb., and the concrete 20¢ per cubic foot. Slab forms have been estimated at 8¢ per sq. ft., and beam and girder forms at 10¢ per sq. ft.

DESIGN NO. 2.

The same general remarks as to the design and the construction of Design No. 1 apply in this design also. The unit prices are the same.

Owing to the increasing cost of formwork, there is a tendency to simplify the framing and eliminate intermediate beams as shown in the first two designs. Architecturally and otherwise a panelled ceiling is objectionable because of its ugly effect. A long span ceiling as shown in the rest of the designs reflects more light and as a result gives a better lighted room than one which is cut up by beams forming pockets which retain the heat and the dust.

DESIGN NO. 3.

The framing shown is the simplest type of a slab and beam design. The required thickness of slab is excessive.

however. Owing to the fact that the neutral axis comes within the slab, the beam was designed as a simple rectangular beam. 1/2" round stirrups closely spaced were required because of the high diagonal tension. Bent slab bars and beam bars were used same as in Design No.1. This same construction was used throughout all the designs in which a solid slab and deep beams were used. Unit costs are The same as in Design No.1.

DESIGN NO. 4.

The assumption that stresses at right angles to each other act independently allows the design of a two way slab. As the panel is square, one half the load is carried in each direction.to the beam. The reduction in the thickness of the slab over the one way slab is considerable. The increased amount of steel and the extra beam in this design are to be compared with the large amount of concrete used in the previous design. The thinner slab in this design made it necessary to design the beam as a T beam. The unit costs are the same as in Design No. 1.

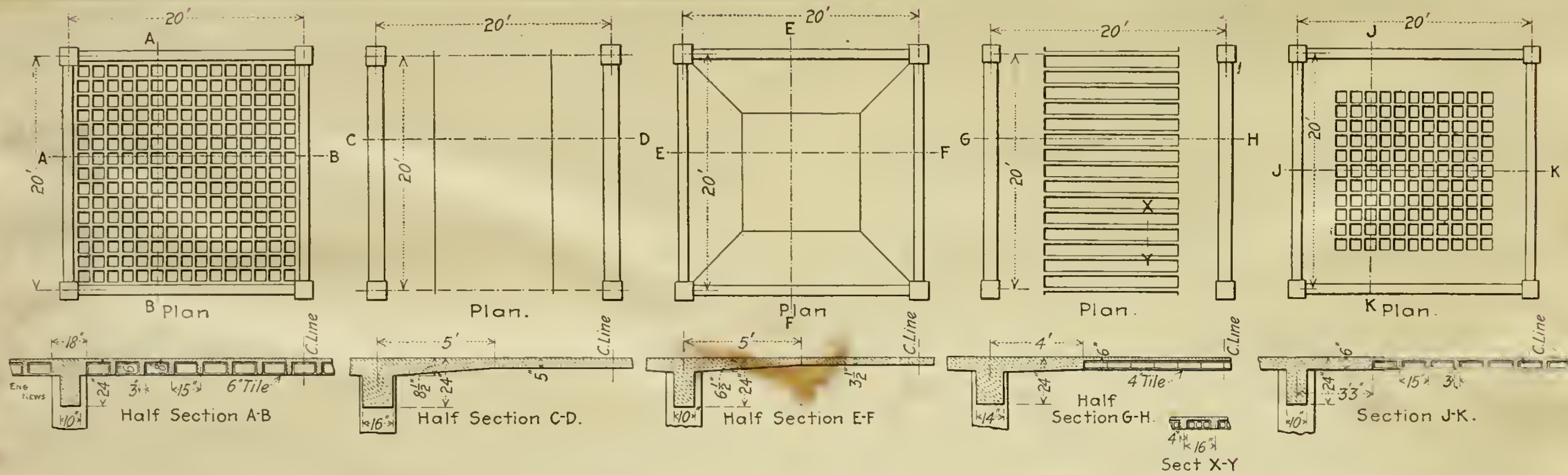
DESIGN NO. 5.

Long span concrete construction has come to stay. The objection to the one-way solid slab construction is its great dead weight. The dead load of the slab shown in Design No.3 is 75% in excess of the load of the floor in this design,and yet the load carrying capacity of the two are the same. Herein is the main argument in favor of the hollow tile floor. There are other advantages, too. The cost of formwork is about one third less than in solid

slab construction. This is because of the fact that it is not necessary to use tongued-and-grooved stuff in the construction of the forms, as all that is required is a 1"x10" board under each concrete joist. There is not the waste of lumber so common with tongued-and-grooved stuff as the 1" x 10" boards are easily placed and easily wrecked. Their salvage value is therefore considerably higher than the lumber for forms where tight sheathing is required.

When a mortar finish is applied immediately after the concrete is poured in this type of a floor, and is left perfectly level, this ~~level~~ surface is not maintained, but there are slight depressions over the joists. This is because of the fact that the two inch top coat dries quicker than the joist with its greater body of concrete and greater shrinkage. The top coat dries quicker because the hollow tile absorbs the moisture in the concrete. To prevent this absorption of moisture by the tile, a through sprinkling of the tile just before the floor is poured should be insisted on, especially in hot weather.

Referring to the section of the floor it will be noticed that it consists of 4" joists with hollow tile fillers between. Over the tile is a 2" layer of concrete, previously designated as the top coat. The hollow tile is first placed in position on the centering, and the joist and top coat poured. The filler merely acts as a form for the joists, and as a convenient surface for plastering. Structurally, it is not considered in the design, and in fact is only so much added dead weight.



Design No. 6.—Two Way Hollow Tile Slab.

Design No. 7.—One Way Cantilever, Solid Slab.

Design No. 8.—Two Way Cantilever, Solid Slab.

Design No. 9.—One Way Cantilever, Hollow Tile Slab.

Design No. 10.—Two Way Cantilever, Hollow Tile Slab.

FIGS. 6-10.

The floor joists are designed as T beams with a 2" thick flange. Two bars are placed in each joist, one straight, and one bent at both ends and extending to the quarter point beyond the center line of the columns in both directions. This last bar is bent up so as to aid in diagonal tension, and also to provide the same steel over the support as in the center of the span.

In any design in which hollow tile floors are used it is necessary to provide enough concrete in compression in the flange of the beam. In this design a 30" flange was required. One half inch stirrups closely spaced were needed because of the heavy diagonal tension characteristic of this type of a design. Beam bars were bent up in the same way as previous designs.

The following unit prices were used in estimating: Steel 3¢ per lb., concrete 20¢ per cubic foot, 8" tile 9¢ per piece, slab forms 6¢ per square foot, beam forms 10¢ per square foot.

DESIGN NO. 6.

Ordinary hollow tile is open at both ends and can not be used when the floor is to be reinforced in both directions. Various two-way tile have been put on the market. Each 15" square shown in this design consists of four separate pieces so manufactured that when they are placed on the centering they present four closed sides to the joists at right angles to each other. The lower edge of each tile projects 1 1/2", giving an unbroken tile surface for plastering.

The principal objection to the two-way hollow tile is that there are four times as many pieces to handle as in ordinary one-way hollow tile floor. The dead weight is about 25% greater than in one-way hollow tile floors of the same total depth.

Half the load was assumed distributed in each direction. The center of gravity of the steel was taken 1 1/2" from the bottom of the floor, and the joist designed as a T-beam. Two bars were put in each joist, one straight and one bent, extending to the quarter point.

The beam was designed as a simple rectangular beam with the usual bent bars and stirrups.

Unit prices adopted for this design are the same as the previous design except that the 6" two-way tile was figured at 11¢ per piece.

DESIGN NO. 7.

In an attempt to produce a more economical construction, the cantilever slab has been adopted rather widely. The method of design is as follows. A flat slab is designed for a span about one-half the c.to c. span. The portion of the slab from the edge of the flat slab to the beam is designed as a simple cantilever beam carrying the reaction from the flat slab at the end and a uniformly distributed load over the cantilever. In this way the required thickness of the slab at the edge of the beam is obtained. The beam is designed to take the reactions from the cantilevers, and also the live and dead load on the beam itself.

The spacing of the bars in the cantilever portion of the slab is so arranged that it is one half c. to c. distance of the slab bars. In this way the slab bars may be bent up near the end of the flat slab, and run in the top of the cantilever slab where this steel is needed. The balance of the steel in the cantilever slab is furnished by straight bars which terminate in the flat slab about one foot from the shallow end of the cantilever. Temperature bars 24 ins c. to c. are used in both the slabs. The arrangement of steel in the beam is the same as in previous designs.

Owing to the more complex slab form construction, the cost per sq. ft. has been estimated at 8 1/2¢. Beam forms have been taken at 10¢ per sq. ft., steel at 3¢ per lb., and concrete at 20¢ per cubic foot.

DESIGN NO. 8.

This design represents a cantilever slab in which the center flat slab is reinforced in both directions, as one-half the load is assumed to be carried in each direction. The cantilever slab has been designed to take the reaction from the two-way slab and also the full uniformly distributed live and dead load on the cantilever itself. As in the previous design the beam has been designed to take the reactions from the cantilevers and also its dead and live load.

The arrangment of steel is similar to Design No.7 except that temperature bars are not used in the flat slab because of the two-way reinforcement.

The method of design results in a reduction in the con-

crete quantities over the previous design,and a considerable increase in the steel quantities. A vaulted ceiling effect is produced which may or may not be desirable. The form work for the slab is rather difficult and has been placed at 9 1/2 per sq. ft. Except for this item the unit prices are the same as in Design No. 7.

DESIGN NO. 9.

The only difference between this and Design No. 7 is that a hollow tile slab has been substituted for a solid slab. The hollow tile slab has been designed as a T beam in the usual way. Two bars were required in each rib, one straight, and one bent up to aid in the cantilever slab. Other features are similar.

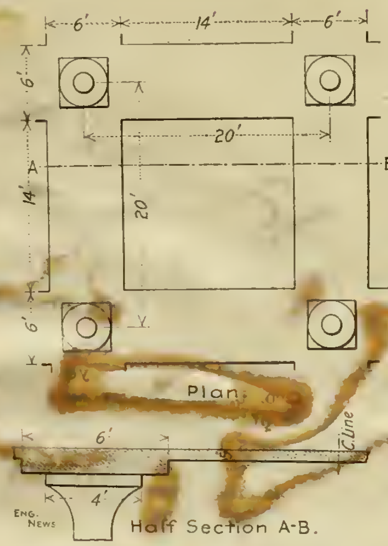
Unit prices are the same as in Design No. 7 with the addition of 5 1/2¢ per piece for the 4-in hollow tile.

DESIGN NO. 10.

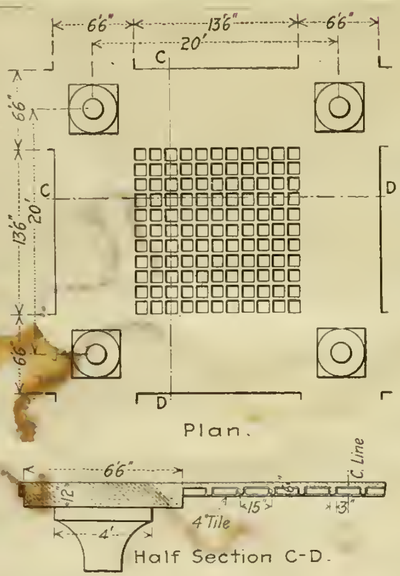
Although this is called a cantilever design the vaulted ceiling of the previous designs is not obtained. This is because of the fact that the depth of the cantilever happens to be come the same as the hollow tile slab. The method of design is similar to Design No. 8, except that the hollow tile floor is designed as a T beam. The arrangement of steel is similar to Design No.9. Steel has been figured at 3¢ per lb., concrete 20¢ per cu. ft., 4" tile 10¢ per piece, slab forms 8¢ per sq. ft., and beam forms 10¢ per sq. ft.

DESIGN NO. 11.

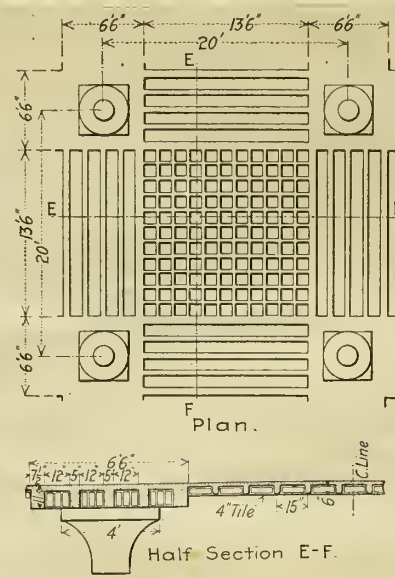
Ordinarily a shallow beam is not economical. In warehouse construction the height to which goods may be piled is limited to the distance between the floor and the under side of the deepest beam of the floor above. In this design there



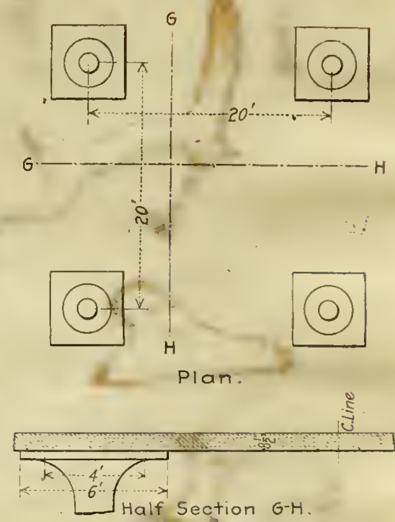
Design No. 11.—Two Way Solid Slab, Shallow Beam.



Design No. 12.—Two Way Hollow Tile Slab, Shallow Beam.



Design No. 13.—Two Way Hollow Tile Slab, Hollow Tile Beam.



Design No. 14.—Flat Slab.

FIGS. 11-14.

is a saving of one foot in head room over the previous design. This means a saving of one foot of brickwork on all exterior walls and also a saving of one foot of column for each floor. Thus a shallow beam design may work out to be economical when the building as a whole is taken, although a typical interior panel may not appear so,

As a preliminery step the width of the shallow beam was assumed. The two-way slab between was designed in the usual way. The shallow beam was designed to take the load from the slab and also the full live and dead load on the beam itself. It was considered as a rectangular beam, and the depth to the center of the steel obtained, since the width was known. Because the diagonal tension was low, no stirrups were used, but one half the beam bars were bentat each end at about the quarter point. To keep these beam bars in position a tie bar was placed at each end of the beam.

No elaborate beam forms are required in this design as the whole panel is practically a flat slab. Steel has been estimated at 3¢ per lb., concrete 20¢ per cu. ft., and slab forms 8 1/2¢ per sq. ft.

DESIGN NO. 12.

This design is indentically the same as the previous one except for the substitution of a hollow-tile slab for a solid slab. Unit prices are the same with the the addition of 10¢ per piece for the 4 in. two-way hollow tile.

DESIGN NO. 13.

A study of the two previous designs discloses the fact that the dead load of the panel is comparatively great. For all designs in which the span and the load of the slab has been the same it has been found that the dead load of the hollow tile floor has been considerably less than the corresponding solid slab floors. Believing that the same general principles could be used in the beam itself, ~~this~~ the following design was originated by the writer.

The slab was designed as in previous designs of this kind. One bar was placed in each joist, straight at one end and resting on the end joist of the hollow tile beam. The other end of the bar was bent up and extended to the quarter point of the 13-ft. 6-in. span. The beam was designed to take the load from the slab, and also the dead load of the concrete and the tile and the live load of the beam itself. The moment thus found was distributed equally on the beam joists, and the joist designed as a T beam. Two bars were placed in each joist, one straight and one bent, extending to the quarter point beyond the edge of the cap in both directions.

Steel was figured at 3¢ per lb., concrete at 20¢ per cu. ft., 4" two-way hollow tile at 10¢ per piece, 8" tile at 9¢ per piece, and slab forms at 8¢ per sq. ft.

DESIGN NO. 14.

Reinforced concrete has an individuality ~~of~~ in itself. To the writer the imitation of standard steel and timber

framing is inconsistent and unnecessary. Why is it necessary to assume that the load travels from the slab into the beam, from the beam into the girder, and from the girder into the column? Tests of full size panels show that the load tends to travel directly to the column. This fact is evidenced by the diagonal cracks which form when a nearly square slab is tested to destruction, and seem to conclusively show that the stress does not travel around a corner as we assume.

The advantages of the flat slab over other forms of construction may be summarized as follows:—

1. Absence of beams with resulting low cost of formwork.
2. Simplicity of reinforcement.
3. Increased available headroom.
4. Ease of installing sprinkler system.

These items will be taken up in the order given.

Lumber is becoming ~~more~~ scarcer and higher in price. The demands of trades unions are such that it is no longer possible to use ordinary labor in the rough part of formwork; but a union carpenter must be employed. So far it has seemed impossible to develop a system in which ~~so that~~ metal forms can be used in a building. For this reason some contractors bid high when there are beams in a job, but will bid low on a girderless floor shown in the design.

The absence of beams means the absence of the troublesome bending and placing of the beam steel. No bending of slab bars is required as long bars of small diameter are used, and these are allowed to sag into the position for which they are designed. The distributing bars of the column

head are bent, but the cost is small.

As pointed out in design No. 11, any increased available headroom means a decided saving when the whole building is involved. The difference of 1'-3 1/2" in headroom between this design and the earlier designs is such a large difference that it may be a determining factor.

It is ~~a~~ difficult to enstall a sprinkler system when the ceiling is cut up with beams. The perfectly flat ceiling of this design makes such an installation extremely simple.

Flat slabs have been studied in research laboratories and also in the field. In both cases the fact has been brought out that the stress ever the support is considerably higher than in the center of the span. In some experiments there has been such a marked difference that it seems that the important thing is to study the stresses at the support, and if these are correctly analyzed the stresses at the middle of the span would be a comparatively simple matter.

There are almost as many opinions as to the design of a flat plate as there are engineers. No two authorities on this subject agree as to the analysis. In the earlier analyses of plates the conclusion was reached that a plate supported on the four corners was stronger than if it were supported on the four edges. On the face of it this is ridiculous. A feeling of distrust in the earlier analyses has been aroused, and a solution sought elsewhere.

Recent tests at the University of Illinois throw considerable light on the correct solution of the problem at hand . At ~~this~~ that institution, footings in which the width of

base was four times the width of the pier, and wide beams which were supported for one half or less of their width, were studied. Apparently these tests have no bearing on the flat slab design; but when it is remembered that the column and its cap can be treated as an inverted pier, the analogy becomes more real.

In a paper read by Mr. Arthur R.Lord, research fellow in the Engineering Experiment Station, University of Illinois, before the National Association of Cement Users, a description of a test of a flat slab was given; and from the information thus obtained various conclusions were arrived at. With these conclusions and the aforesaid mentioned tests in mind an analysis and design of a flat slab was obtained. The method was as follows. A line of inflection was assumed. The square within the line of inflection was considered as an inverted footing with a uniformly distributed load and also with a load along the lines of inflection. Assuming a depth to the center of the steel and knowing the bending moment, the required area of steel was found in the usual way.

Owing to the very small amount of bending, steel was figured at 2 3/4¢ per lb. Concrete was estimated at 20¢ per cu. ft., and formwork at 8¢ per sq. ft.

SUMMARY OF QUANTITIES AND COSTS.

A table showing the quantites for each design is given herewith. The steel quantites represent all the slab and beam steel in the panel. The concrete quantities include everything in the panel except the column cap which is considered as part of the column. As stated before slab forms

TABLE GIVING SUMMARY OF DESIGNS OF VARIOUS TYPES OF REINFORCED-CONCRETE SLABS.

Quantities.

Design No.	Steel lbs.	Concrete cu. ft.	Tile No. pcs.	Forms: Slab sq. ft.	Forms: Beam sq. ft.	Panel dead load.
1	1,550	187	...	400	254	35,300
2	1,837	225	...	400	225	40,900
3	1,976	346	...	400	68	59,100
4	2,251	261	...	400	142	46,400
5	1,597	184	262	400	68	41,600
6	1,905	199	225	400	130	46,100
7	1,587	232	...	400	74	42,000
8	1,800	201	...	400	142	37,400
9	1,386	181	180	400	71	37,400
10	1,457	207	121	400	142	41,700
11	1,778	286	...	400	...	50,200
12	1,678	270	121	400	...	51,100
13	1,689	180	121-4-in. 108-8-in.	400	...	40,400
14	1,110	284	...	400	...	49,800

COST PER SQUARE FOOT.

Design No.	Steel. cts.	Concrete. cts.	Tile. cts.	Forms. cts.	Total. cts.
1	11.6	9.3	...	14.4	35.3
2	13.8	11.2	...	13.6	38.6
3	14.8	17.3	...	9.7	41.8
4	16.9	13.0	...	11.6	41.5
5	12.0	9.2	5.9	7.7	34.8
6	14.4	9.9	6.1	9.3	39.7
7	11.9	11.6	...	10.3	33.8
8	13.5	10.0	...	13.1	36.6
9	10.4	9.1	2.4	10.3	32.2
10	10.9	10.3	3.0	11.6	35.8
11	13.4	14.3	...	8.5	36.2
12	12.6	13.5	3.0	8.5	37.6
13	12.7	9.0	5.3	8.0	35.0
14	7.6	14.2	...	8.0	29.8

include the area within the center lines columns, and beam forms are measured by the square foot of surface in contact with the concrete. These quantities for a typical panel are given as it is realized that the unit prices in other localities may vary from those given with the various designs.

The last column gives the panel dead load. This includes the weight of the construction, and also the weight of the plaster and finish for the panel. This last item is a constant quantity for each design. The panel dead loads were inserted to show the relative dead load brought to the columns and footings by each design. As will be noted there is as wide a variation as in the quantities themselves.

A summary of the cost per square foot is given in the last table. This cost per square foot is the cost for the panel only, and does not include the cost of the columns or the footing for the typical panel. When the building as a whole is considered the total costs given in the last column will not necessarily indicate the relative cost of each design. Other factors may enter in which would materially change these relations. But it is believed that this last column indicates fair relative costs when the concrete only is considered, for average conditions of labor and material in the Middle West.

CONCLUSIONS.

A study of the total costs given in the above table discloses the fact that in every case the hollow tile design is cheaper than the similar solid slab design. The most uneconomical design is that in which a long span solid

slab is supported on beams on one side of the panel only. A close second to this design is ~~that~~ the one in which a two-way solid slab is supported by beams on all sides of the panel.

A question is sometimes raised as to the relative economy of one-way and two-way hollow tile. By comparing Design No.5 with No.6, and Design No. 9, with No.10, it is seen that there is a marked difference in favor of the one-way hollow tile design. The two-way hollow tile is in turn more economical than the corresponding solid slab design.

It has been stated that a shallow beam is not ecomomical. By comparing Design No. 4 with Design No.11, and Design No.6 with Design No. 12, it is seen that the shallow beam is considerably cheaper than the deep beam design. Of all the designs the flat slab appears to be the most economical. Of the three shallow beam designs, the design in which hollow tile is used throughout, works out to be the most economical.

CHAPTER IV.

Concrete Column Economics.

In order to test the truth of the frequently quoted statement that "the cheapest reinforcement is cement", the writer has designed a number of different types of reinforced concrete columns under typical conditions of load, and has made detailed comparisons of their cost.

All columns were designed in conformity with the present Building Ordinance of the City of Chicago. The sections of this ordinance relating to reinforced concrete columns are as follows:

Ratio of Moduli of Elasticity—Adhesion—Bond. The calculations for the strength of reinforced concrete shall be based on the assumed ultimate compressive strength per square inch designated by the letter "U" given in the table below for the mixture to be used.

The ratio designated by the letter "R" of the modulus of elasticity of steel to that of the different grades of concrete shall be taken in accordance with the following table:

Mixture	U	R
1 cement, 1 sand, 2 broken stone, gravel or slag......	2,900	10
1 cement, 1½ sand, 3 broken stone, gravel or slag......	2,400	12
1 cement 2 sand, 4 broken stone, gravel or slag......	2,000	15

The compressive stress in steel shall not exceed the product of the compressive stress in the concrete multiplied by the elastic modulus of the steel and divided by the elastic modulus of the concrete.

Direct compression in concrete shall be one-fifth of its ultimate strength.

All reinforcing steel shall be accurately located in the forms and secured against displacement, and inspected by the representative of the architect or engineer in charge before any surrounding concrete be put in place. It shall be afterwards completely inclosed by the concrete, and such steel shall nowhere be nearer the surface of the concrete than 1½ in. for columns.

Reinforced Concrete Columns—Limit of Length—Per Cent. of Reinforcement—Bending Moment in Columns—Tying Vertical Rods. (a) Reinforced concrete may be used for columns in which the concrete shall not be leaner than a 1: 2: 4 mixture, and in which the ratio of length to least side or diameter does not exceed 12, but in no case shall the cross-section of the column be less than 64 sq.in. Longitudinal reinforcing rods must be tied together to effectively resist outward flexure at intervals of not more than 12 times least diameter of rod and not more than 18 in. When compression rods are not required, reinforcing rods shall be used, equivalent to not less than ½% of the cross-sectional area of the column; provided, however, that the total sectional area of the reinforcing steel shall not be less than 1 sq.in., and that no rod or bar be of smaller diameter or least dimensions than ½ in. The area of reinforcing compression rods shall be limited to 3% of cross-sectional area of the column. Vertical reinforcing rods shall extend upward or downward into the column, above or below, lapping the reinforcement above or below enough to develop the stress in rod by the allowable unit for adhesion. When beams or girders are made monolithic with or rigidly attached to reinforced-concrete columns, the latter shall be designed to resist a bending moment equal to the greatest possible unbalanced moment in the beams or girders at the columns, in addition to the direct loads for which the columns are designed.

When the reinforcement consists of vertical bars and spiral hooping, the concrete may be stressed to one-fourth of its ultimate strength, provided, that the amount of vertical reinforcement be not less than the amount of the spiral reinforcement, nor greater than 8% of the area within the hooping; that the percentage of spiral hooping be not less than ½% nor greater than 1½%; that the pitch of the spiral hooping be uniform and not greater than one-tenth of the diameter of the column, nor greater than 3 in.; that the spiral be secured to the verticals at every intersection in such a manner as to insure the maintaining of its form and position, that the verticals be spaced so that their distance apart, measured on the circumference be not greater than 9 in., nor one-eighth the circumference of the column within the hooping. In such columns, the action of the hooping may be assumed to increase the resistance of the concrete equivalent to 2½ times the amount of the spiral hooping figured as vertical reinforcement. No part of the concrete outside of the hooping shall be considered as a part of the effective column section.

It will be noticed that the ratio of the modulus of the steel to that of concrete varies with the richness of the mix. It seems to the writer that this is the logical way, instead of using the same value for all mixes, as is sometimes done. The requirements in regard to materials and workmanship are such as to insure the best grade of concrete structures.

In the computations of these columns, certain formulas were used. The first of these is for what the writer calls "reinforced concrete columns", that is, for columns reinforced with vertical steel only, tied in a suitable manner. This formula is the same as found in standard textbooks on reinforced concrete. Its derivation is only repeated for the sake of clearness, and to bring out the derivation of the formula for hooped columns. By "hooped columns" is meant columns reinforced with vertical steel and with spiral hooping also.

NOTATION.

It was necessary in the derivation of these formulas to adopt the following notation:

f = Average allowable unit compression upon the effective cross-section of the column.

f_c = Allowable unit compression upon the concrete of the column.

f_s' = Allowable unit compression upon the vertical steel in the column.

$n = \frac{E_s}{E_c}$ = Ratio of the modulus of elasticity of steel to modulus of elasticity of concrete.

P = Load to be carried by the column.

A = Area of total effective cross-section of the column = area out to out of hooping in hooped column = net sectional area of reinforced concrete column.

A_s = Area of vertical steel in cross-section.

A_c = Area of concrete in cross-section.

A_h = Area of hooping steel in cross-section.

$p = \frac{A_s}{A}$ = Ratio of cross-section of vertical steel to effective cross-section of column.

$p' = \frac{A_h}{A}$ = Ratio of cross-section of hooping steel to effective cross-section of column.

Formulas.

Below is given the derivation of the various formulas.

Reinforced Concrete Columns. The ordinance states that the compressive stress in vertical steel shall not exceed $f_c n$. Therefore $f_s' = nf_c$. Total load carried by steel $= f_s' A_s = nf_c A_s$. Total load carried by concrete $= f_c A_c$. Total load carried by column

$$= P = nf_c A_s + f_c A_c. \qquad P = fA, \text{ and } A_c = A - A_s$$

therefore $fA = nf_c A_s + f_c(A - A_s)$

$$f = f_c\left[\frac{nA_s}{A} + \left(1 - \frac{A_s}{A}\right)\right], \text{ but } \frac{A_s}{A} = p, \text{ therefore}$$

$$f = f_c\left[np + (1-p)\right] = f_c\left[1 + (n-1)p\right] \qquad (1).$$

Hooped Columns. The ordinance states " the action of the hooping may be assumed to increase the resistance of the concrete equivalent to two and one-half times the amount the amount of the spiral hooping

figured as vertical reinforcement".

Total load carried by hooping = $2.5\ f_s'\ A_h = 2.5\ nf_cA_h$

"increased resistance of concrete" = $\dfrac{2.5nf_cA_h}{A} = 2.5nf_cp'$

Total allowable resistance of concrete

$= f_c + 2.5\ nf_cp' = f_c(\ 1 + 2.5\ np'\ ).$

Total load carried by concrete = $f_c(\ 1 + 2.5\ np')\ A_c$

Total load carried by steel $= f_c(\ 1 + 2.5\ np')nA_s$

Total load carried by hooped column = fA =

$f_c(\ 1 + 2.5\ np'\ )\ (\ A_c + nA_s\ )$. But $A_c = (\ A - A_s - A_h)$, and

therefore $fA = f_c(\ 1 + 2.5\ np')(A - A_s - A_h + nA_s)$

$= f_c(\ 1 + 2.5\ np')\ (\ A - A_h + (\ n-1\ )\ A_s).$

But $\dfrac{A_h}{A} = p'$, and $\dfrac{A_s}{A} = p$. Therefore

$$f = f_c(\ 1 + 2.5\ np'\ )\left[1 - \frac{A_h}{A} + (\ n - 1\ )\frac{A_s}{A}\right]$$

$$= f_c(1 + 2.5\ np'\ )\left[1 - p' + (\ n-1\ )\ p\right] \qquad (\ 2\ ).$$

The formulas thus derived are too unwieldy for ordinary use. In order to facilitate the computations of the columns, these formulas were plotted on four separate diagrams. Diagram No.1 (Fig. 1 p 50) shows the relation between the percentage of vertical steel in a reinforced concrete column of the concrete mixes shown, and the average allowable compression. Diagrams Nos. 2,3,and 4 (Figs. 2,3,and 4 p 50) show the relation between the percentage of vertical steel and the average allowable unit compression with the spiral reinforcement and the mix known. The extremities of each diagram represent the minimum and maximum percentages of vertical steel allowed by the ordinance, and the top and bottom

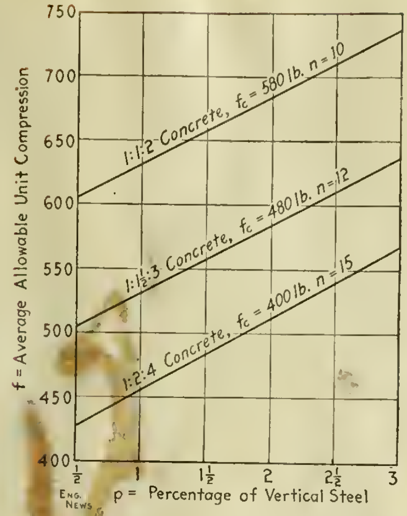
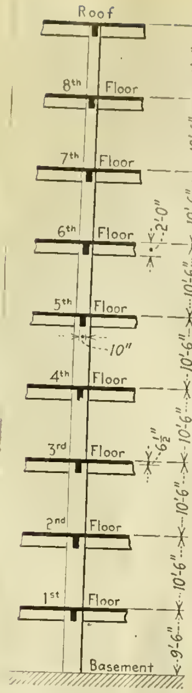


FIG. 1. VERTICALLY REINFORCED-CONCRETE COLUMNS. GRAPHS OF THE FORMULA $f = fc\ [1 + (n - 1)\ p]$



Column Schedule.

8th.	L.L.	400x25	10,000
	D.L.		30,300
	Col.		1,500
		Total,	41,800
7th.	L.L.	400x100x0.85	34,000
	D.L.		46,000
	Col		2,200
		Total,	124,000
6th	L.L.	400x100x0.80	32,000
	D.L.		46,000
	Col.		3,000
		Total,	205,000
5th	L.L.	400x100x0.75	30,000
	D.L.		46,000
	Col.		4,000
		Total,	285,000
4th.	L.L.	400x100x0.70	28,000
	D.L.		46,000
	Col.		5,000
		Total,	364,000
3rd.	L.L.	400x100x0.65	26,000
	D.L.		46,000
	Col.		6,000
		Total,	442,000
2nd.	L.L.	400x100x0.60	24,000
	D.L.		46,000
	Col.		7,000
		Total,	519,000
1st.	L.L.	400x100x0.55	22,000
	D.L.		46,000
	Col.		8,000
		Total,	595,000
Bsm.	L.L.	400x100x0.50	20,000
	D.L.		46,000
	Col.		9,000
		Total,	670,000

FIG. 5. OUTLINE OF COLUMN SHAFT ANALYZED FOR COSTS OF COMPARATIVE TYPES OF COLUMN

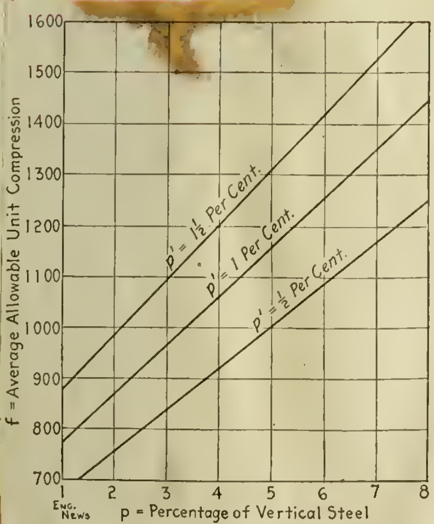


FIG 2. 1:2:4 CONCRETE. $n = 15$, $fc = 500$

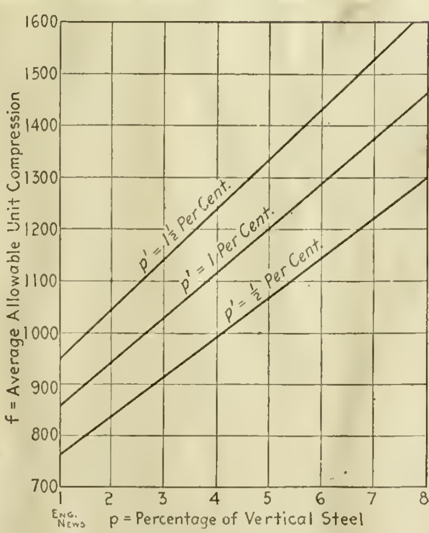


FIG. 3. 1:1½:3 CONCRETE. $n = 12$, $fc = 600$

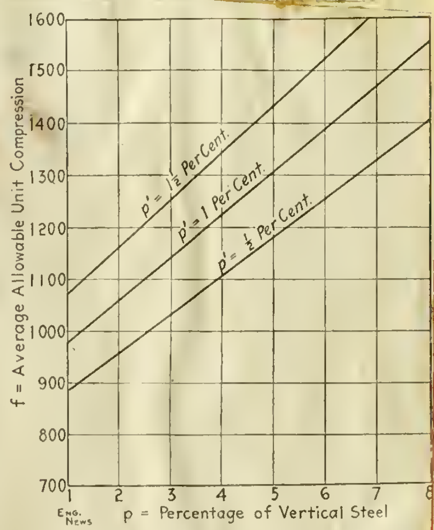


FIG. 4. 1:1:2 CONCRETE. $n = 10$, $fc = 725$

FIGS. 2-4. HOOPED REINFORCED-CONCRETE COLUMNS
GRAPHS OF THE FORMULA: $f = fc\ [1 + 2.5\,np']\ [1 - p' + (n - 1)\ p]$

curves represent the maximum and minimum percentages of spiral reinforcement allowed.

It will be seen by a study of these curves that some of these values for the allowable u n it compression run extremely high. A comparison of some of these values ~~and of~~ with laboratory tests in which the mix, the percentage of vertical steel, and of spiral steel are identical, discloses the fact that the factor of safety based on the yieldpoint of the column is about 2.5. For all practical purposes the yieldpoint represents the ultimate strength of the column, as the resulting deformation after the yield point is passed is so great that the reinforced concrete structure would be badly cracked, if not wrecked, if this deformation such occur.

Wishing to satisfy himself as to this question of economical design, the writer designed a column shaft. A panel 20 x 20 ft. was taken. The building was assumed to be eight stories high, and the column designed for a roof load of 25 lb. per sq. ft. and a floor load of 100 lbs. per sq. ft. In carrying down the live load on the floors, 85% was taken on the top floor, and a reduction of 5% of the live load taken for each floor until 50% was reached, when no further reduction was made. It was assumed that the floors of this building could not all be loaded with the full live load at once, and that such a reduction was permissible according to the ordinance. A typical section of the panel and a column schedule is given in Fig. 5. The dead load given includes

the weight of the construction, the weight of the granitoid finish, and an allowance for partitions.

In the original study an investigation was made of the columns on the seventh, fifth, third, and first floors, thus insuring a range of total loads sufficiently great ~~so that~~ to obtain an idea ~~could be obtained~~ of the relative economy for various loads. On account of lack of space it is impossible to present all these figures here, but it is thought that the accompanying tables I - IV p 50 will be sufficient to demonstrate the extent of the investigation. Table I shows the complete analysis of the vertically reinforced concrete column for the fifth story, under a load of 285,000 lbs. Table II shows the complete analysis of the same column for the hooped column, with 1/2% of spiralling. For both types three different percentages of vertical reinforcing and three different mixtures of concrete were investigated. Table III gives the results of the computations shown in Table I for the four separate load under investigation, and Table IV the results of the computations shown in Table II for the same loads. Each of the ~~figures~~ values given in Tables III and IV were computed in the manner outlined in Tables I and II and described in detail below.

Method of Design and Explanation of Tables.

In the design of hooped columns the following method was used. Knowing the total load, a concrete mixture, percentage of spiral steel p', and percentage of vertical steel p was assumed. With the aid of Diagrams 1,2,3,

p. 50

and 4 an average allowable unit compression f was obtained from the left of the diagram. The " Total Load" P (Fig. 5) was divided by this value of f, thus obtaining the " Area of Core"(Table II). The "Area of Vertical steel" A_s represents the product of A and p. Knowigg A, the "Dia. of Core" D was obtained from tables, to the nearest inch. The " Size of Col." represents the side of a square column, or the short diameter of an octagonal column, or the diameter of a round column. It was obtained by adding 3 in. to the diameter of the core, as the ordinance requires 1-1/2 in. fireproofing of the core.

The ordinance states that " The spiral hooping shall be figured as vertical reinforement". In order to obtain the "Dia. & Pitch of Spiral", it was necessary to use the following formula.

Let s = Pitch of spiral in inches,

D = Diameter of core of spiral in inches,

a = Area of cross-section of spiral,

Then

$$A_h = \frac{3.14\ a\ D}{s} .$$

A_h equals the product of A and p'. Assuming a, and knowing D and A_h, the pitch was figured from the above formula. If the pitch of the spiral became greater than that allowed by the ordinance, a smaller diameter spiral was used. The pitch is given to the nearest 1/8 in., as the mills that fabricate spirals can manufacture them to such a pitch.

TABLE I. ANALYSIS OF COST OF VERTICALLY REINFORCED-CONCRETE SQUARE COLUMNS, UNDER LOAD OF 285,000 LB.

Description	1:2:4 Concrete Vertical Steel 1%	2%	3%	1:1½:3 Concrete Vertical Steel 1%	2%	3%	1:1:2 Concrete Vertical Steel 1%	2%	3%
Area of core	625	556	502	535	488	447	450	417	387
Area of vert. steel	6.25	11.12	15.06	5.35	9.76	13.41	4.50	8.34	11.61
Size of sq. col	28″	27″	26″	26″	25″	24″	24″	24″	23″
Wt. of vert. steel	274	503	658	233	421	594	210	368	503
Wt. of ties	16	15	15	15	14	14	14	14	13
Cu.ft. of concrete	54.5	50.8	47.0	47.0	43.5	40.0	40.0	40.0	36.8
Total cost	27.99	33.22	36.40	25.63	29.61	33.29	23.94	28.26	30.85

TABLE II. ANALYSIS OF COST OF SPIRALLY HOOPED REINFORCED-CONCRETE COLUMNS UNDER LOAD OF 285,000 LB. AND WITH ½% SPIRALS

Description	1:2:4 Concrete Vertical Steel 2%	4%	6%	1:1½:3 Concrete Vertical steel 2%	4%	6%	1:1:2 Concrete Vertical steel 2%	4%	6%
Area of core	376	308	262	340	288	250	298	258	228
Area of vert. steel	7.52	12.32	15.72	6.80	11.52	15.00	5.96	10.32	13.68
Dia. of core	22″	20″	19″	21″	19″	18″	20″	18″	17″
Size of col	25″	23″	22″	24″	22″	21″	23″	21″	20″
Dia. and pitch of spiral	¼″-1¾″	¼″-2″	¼″-2¼″	¼″-1⅞″	¼″-2″	¼″-2¼″	¼″-2″	¼″-2″	¼″-2″
Wt. of vert. steel	342	548	684	289	499	658	263	447	605
Wt. of spiral steel	69	55	47	62	52	44	55	50	47
Cu.ft. conc. sq. col	43.5	36.8	33.6	40.0	33.6	30.7	36.8	30.7	27.8
Cu.ft. conc. oct. col.	36.0	30.5	27.8	33.1	27.8	25.4	30.5	25.4	23.0
Cu.ft. conc. round col	34.1	28.9	26.4	31.4	26.4	24.1	28.8	24.1	21.8
Total cost sq. col	28.78	31.93	34.30	26.83	30.10	33.22	25.92	28.58	31.75
Total cost oct. col	28.35	31.67	34.14	26.30	29.82	32.96	25.37	28.15	31.35
Total cost round col	30.54	33.54	36.09	28.35	31.73	34.76	27.13	29.92	33.18

TABLE III. COMPARATIVE COST IN DOLLARS OF VERTICALLY REINFORCED-CONCRETE SQUARE COLUMNS UNDER FOUR DIFFERENT LOADS

Total loads, lb.		1:2:4 Concrete Vertical steel 1%	2%	3%	1:1½-3 Concrete Vertical steel 1%	2%	3%	1:1:2 Concrete Vertical steel 1%	2%	3%
124,000	Side of sq. col., in	20	19	18	19	18	17	17	17	16
	Cost in $	15.67	17.35	18.59	14.66	16.15	18.08	13.94	15.79	16.71
285,000	Side of sq. col., in	28	27	26	26	25	24	24	24	23
	Cost in $	27.99	33.22	36.40	25.63	29.61	33.29	23.94	28.26	30.85
442,000	Side of sq. col., in	34	33	31	32	31	29	30	29	28
	Cost in $	40.13	48.02	53.39	37.47	44.08	48.99	34.73	40.77	45.64
595,000	Side of sq. col., in	39	37	36	37	35	34	34	33	32
	Cost in $	51.58	62.11	71.90	48.15	58.01	65.99	43.88	53.46	60.63

TABLE IV. COMPARATIVE COST IN DOLLARS OF SPIRALLY HOOPED COLUMNS

Under Load of 285,000 Lb.

[Variation of Concrete Mix, Vertical Steel and Spiralling.]

Vertical Steel		1:2:4 Concrete 2%	4%	6%	1:1½:3 Concrete 2%	4%	6%	1:1:2 Concrete 2%	4%	6%
½% Spiral	Square	28.78	31.93	34.30	26.83	30.10	33.22	25.92	28.58	31.75
	Octagon	28.35	31.62	34.14	26.30	29.82	32.96	25.37	28.15	31.35
	Round	30.54	33.54	36.09	28.35	31.73	34.76	27.13	29.92	33.18
1% Spiral	Square	28.15	30.58	31.70	27.02	29.42	31.55	25.78	27.92	30.28
	Octagon	27.75	30.42	31.52	26.64	29.16	31.29	25.32	27.50	30.11
	Round	29.86	32.37	33.42	28.48	30.96	33.15	27.20	29.34	31.71
1½% Spiral	Square	26.84	29.40	29.64	26.57	28.53	30.13	25.58	28.15	30.23
	Octagon	26.68	29.24	29.68	26.30	28.26	30.10	25.13	27.75	30.08
	Round	28.62	31.06	31.34	28.21	30.12	31.72	26.88	29.57	31.66

In computing the "Wt. of Vert. Steel", the bars were assumed long enough to lap 30 diameters, and the weight obtained on that basis. The "Wt. of Spiral Steel," was obtained by multiplying A_h by 3.4, and also by the length of the column. The weight of the spacers used in this type of reinforcement is not included, but an allowance made for it in the unit price used.

The " Cu. Ft. Conc." for square, octagon, and round columns includes the concrete from the floor line to the under side of the slab above.

An item which does not appear in the table is that of formwork. This formwork is measured by ~~by~~ the square feet of surface in contact with the column. Thus a 24 in. square column, 10 ft. high, would have 4' x 2' x 10' = 80 sq. ft. of formwork per column.

The " Total Cost" was obtained by adding together the different items of vertical steel, spiral steel, concrete, and formwork for the column in question.

Reinforced concrete columns were designed in the same general way. Ties, 1/4 in. round, 12 in. c. to c., were used throughout. The weight and cost of these are included in Table I.

The question may be asked why it is necessary to obtain the total cost of square, octagon, and round columns. It would seem that the round column would be the cheapest since it has the least amount of concrete in it. In all three columns the vertical and spiral

steel is the same; But th~~is~~e ~~element of~~ formwork greatly effects the cost, as will be shown later.

Discussion of Costs.

The analysis of the cost of a steel column is an easy matter compared with that of a reinforced-concrete column. In the latter column, four variables, vertical steel, spiral steel, concrete, and formwork must be accounted for in the total cost.

Vertical steel was figured at $ 1.35, per 100 lbs., base, f.o.b. Pittsburgh in car load lots, with an 18¢ freight rate to Chicago. ~~The above is the price per 100 lbs~~. The cost of unloading from the cars, hauling to the job, bending, and placing has been estimated, ~~so that~~ and the cost in place ~~can be~~ is put at 2- 3/4¢ per lb. Spiral steel has been estimated at $ 60 per ton, f.o.b. Chicago, and its cost in place figured at 4¢ per lb. This cost includes the placing of spacers, and the tying of the vertical steel to the spirals. The cost of material, bending, and placing of the ties used in the reinforced concrete columns has been estimated at 4¢ per lb. It is assumed that the steel can be handled by laborers.

Cement has been estimated at $ 1.25 per bbl., sand $ 1 per cu. yd., crushed stone $ 1.65 per cu. yd., all f. o. b. the job. The cost of mixing, placing, incidental expenses of mixing and hoisting plant, has been estimated at $ 1. 60 per cu. yd. of concrete.

Since the labor of mixing and placing a lean mixture is the same as a rich mixture, it follows that the

only difference is in the cost of the materials. On the assumption that laborers can be obtained at 37-1/2¢ per hour, the cost per cubic foot in place of tho three mixtures has been placed as follows:

1:2:4 concrete	21¢
1:1-1/2:3 ,,	23¢
1:1:2 ,,	26¢

In the estimating of formwork a square column has been taken as a standard. It is assumed that yellow pine will be used in the forms, the price per thousand being taken at $ 22. The wages of form carpenters has been taken at 60¢ per hour. The cost per lineal foot of an octagonal column has been estimated at 15% more than a square column of the same size, ~~while~~ and that of a round column at 50% more than a square column of the same width. The costs given include the cost of the lumber used, the labor of wrecking, the labor of erecting, the labor of ripping to a smaller size, and the labor of erecting and wrecking again. It is assumed that each floor is typical. With the above information in mind the cost per square foot of surface has been estimated as follows:

Square column, below 18 in.	10¢
Square column, 18 to 36 in.	9¢
Octagon column, 18 to 36 in.	12-1/2¢
Round column, 18 to 36 in.	17¢

Perhaps the clearest way to show how these costs were obtained is to carry the analysis of one column through. Assume a 1:2:4 mixture with 1/2% spiral steel

and 4¢ vertical steel. Assume a total load of 285,000 lb. Table II shows that ~~that~~ the size of the octagon column required is 23 in. For a column 10 ft. high, the area ~~square feet~~ of formwork is 10 x 0.276 x 23 = 63.5 sq.ft. The other quantities of steel and concrete will be found in the table. The items of cost will appear as follows:

Vertical Steel, 548 lbs. at 2-3/4¢	=	$ 15.07
Spiral steel, 55 lb. at 4¢	=	2.20
Concrete, 30.5 cu. ft. at 21¢	=	6.41
Forms, 63.5 sq. ft. at 12-1/2¢	=	7.94
Total		$ 31.62

Conclusion.

It is realized that the unit prices in different localities may be different than those given by the writer. For this reason the quantities for each column are given so that a comparison may be made. With the information available the following conclusions were arrived at.

Taking up the columns reinforced with vertical steel only, it will be noticed that for the same load and the same mix the addition of vertical steel in every case is decidedly uneconomical. For the same load and the same percentage of vertical steel, the cheapest column is that which employs the richest mix. In every case the cheapest column for any load is that which has the least amount of vertical steel, and the richest mix.

The same general remarks as to vertical steel and richness of mix applies in the case of hooped columns when the percentage of spiral reinforcement is the same. For the same load, the same mix, the same percentage of vertical steel, but ~~of~~ different percentage of spiral steel, the column with the lowest percentage of spiral steel is the cheapest. Comparing the cost of square, octagan, and round columns having ~~of~~ the same load, the same mix, and the same percentages of vertical and spiral steel, the octagon column is the cheapest. This octagon column has the further advantage that it looks considerably smaller than a square column, and almost as small as a round column of the same diameter. For the heaviest load the cheapest hooped column is an octagon column of the richest mix and the lowest percentage of vertical and spiral steel.

Comparing the column with vertical steel only with the hooped column it will be seen that for the same load the most economical hooped column is always more expensive than the most economical reinforced concrete column. Taking the series as a whole it is evident that, " the cheapest reinforcement is cement".

CHAPTER V.

Comparative Designs of Plain and Reinforced Concrete Column Footings.

A question often raised in the design of column footings, is that of the comparative costs of plain and reinforced concrete footings. In most designs, for the same load and soil pressure, the amount concrete is about the same; but the plain concrete footing is much deeper than the reinforced concrete footing, while it has the advantage that no steel is required. The economics of the question will be taken up in this chapter ~~the following paragraph.~~

PLAIN CONCRETE FOOTINGS.

By plain concrete footing is meant the ordinary stepped footing, unreinforced. A rule commonly used in designing the steps or offsets is that the length of the offset should be one-half of its depth. This rule of thumb, in common with other such rules, does not make an allowance for variations in soil pressure, and for this reason is defective. There would be the same proportions, according to the above rule, for a footing designed for 3500 lbs. per square foot, as for a footing designed for 4500 lbs. per square foot.

The first value for unit soil pressure given above is the value commonly used in designing foundations resting on the ordinary clay found in the City of Chicago, while the second value is that used for a clay which is dry and thoroughly compressed.

A rational method for obtaining the correct

proportion of length to depth of offset for any soil pressure is given in the following analysis. 9 A column footing as shown in Fig.1 p.61 is considered as a series of projecting cantilevers subjected to a uniform soil pressure. The bending moment and the resisting moment of any offset is equated, and a formula (1) p.62 obtained giving the proportion of length to depth as a function of the soil pressure and the allowable tension in the concrete cantilever. This tension, or extreme fiber stress,has been taken as 30 lbs. per sq.in., as this value was considered reasonably conservative for a 1:2:4 concrete.On the basis of formula (1), values for two different soil pressures are given in formulas (2) and (3).

REINFORCED CONCRETE FOOTINGS.

The ~~whole~~ design of the reinforced concrete footings was based ~~in general~~ on the recommendations ~~as to the design of footings given~~ in the University of Illinois Bulletin No.67, entitled" Reinforced Concrete Wall Footings and Column Footings," by A.N.Talbot. The unit stresses were those given in the Revised Chicago Building Ordinances. The steel was designed so that the unit tensile stress did not exceed 18000#, and the bond stress 100# per sq.in. The depth of all footings was such that the punching shear measured on a section of the beam between the center of action of the horizontal forces was not greater than 133# per sq.in.

Plain Concrete Footings.

Proportion of Length to Depth of Offset.

Uniform Load= w

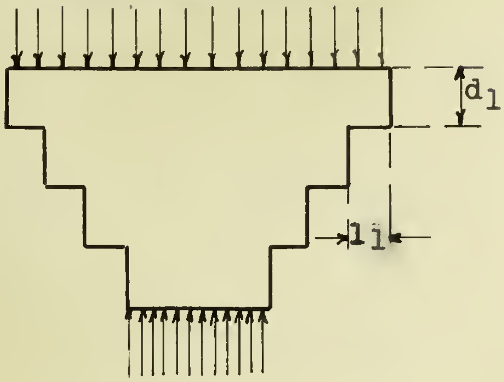


Fig.1.

Let w = allowable unit soil pressure in lbs. per sq.ft.

,, l_1 = length of offset,in inches.

,, d_1 = depth ,, ,, ,, ,, .

,, b = width ,, ,, ,, ,, , = 12".

,, f = extreme fiber stress in lbs. per sq.in.

,, M = bending moment in inch-lbs.

,, R.M.=resisting ,, ,, ,, ,,.

Derivation of Formula.

$$M = \frac{1}{2} w \left(\frac{l_1}{12}\right)^2 \times 12 = \frac{w l_1^2}{24}, \quad R.M. = \frac{1}{6} f b d_1^2 = \frac{1}{6} f\ 12\ d_1^2 = 2 f d_1^2$$

$$M = R.M., \quad \frac{w l_1^2}{24} = 2 f d_1^2, \text{ therefore } l^2 = \frac{48 f d l^2}{w}$$

$$l_1 = \sqrt{\frac{48 f d_1^2}{\sqrt{w}}} = d_1 \sqrt{\frac{48 f}{\sqrt{w}}} \qquad (1)$$

When f = 30#, w = 3500#, then l_1 =.64 d_1 (2)

,, ,,= ,, , ,,= 4500#, ,, l_1 =.56 d_1 (3).

Plain Concrete Footings.

Proportion of Length to Depth of Offset.
Uniform Load= w

d_1

l

Fig.1.

Let w = allowable unit soil pressure in lbs. per sq.ft.

" l_1 = length of offset, in inches.

" d_1 = depth " " " " .

" b = width " " " " , = 12".

" f = extreme fiber stress in lbs. per sq.in.

" M = bending moment in inch-lbs.

" R.M.=resisting " " " ".

Derivation of Formula.

$$M = \frac{1}{2} w \left(\frac{l_1}{12}\right)^2 x\ 12 = \frac{w l_1^2}{24}, \quad R.M. = \frac{1}{6} f b d_1^2 = \frac{1}{6} f\ 12\ d_1^2 = 2 f d_1^2$$

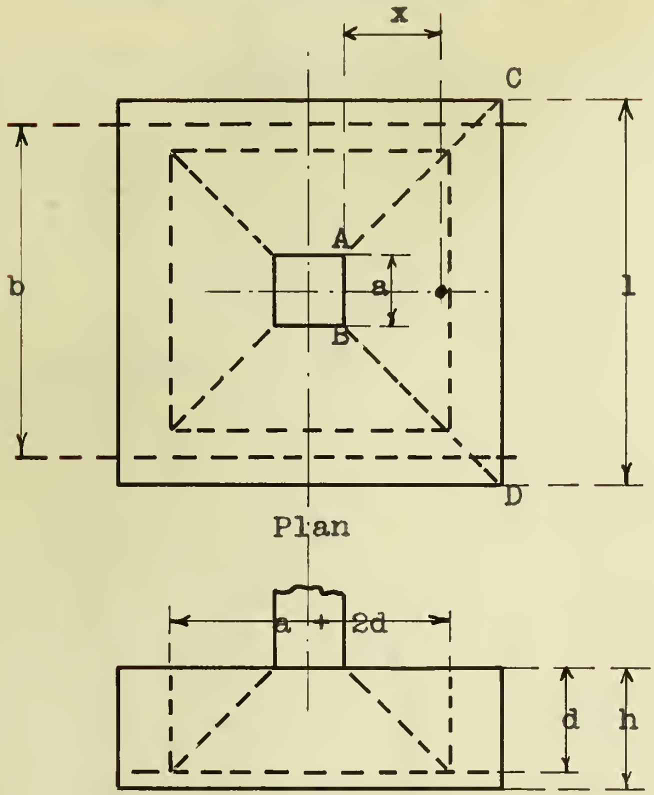
$$M = R.M., \quad \frac{w l_1^2}{24} = 2 f d_1^2, \text{ therefore } l_1^2 = \frac{48 f d_1^2}{w}$$

$$l_1 = \frac{\sqrt{48 f d_1^2}}{\sqrt{w}} = d_1 \frac{\sqrt{48 f}}{\sqrt{w}} \qquad (1)$$

when f = 30#, w = 3500#, then l_1 =.64 d_1 (2)

" " " = " , " " = 4500#, " l_1 =.56 d_1 (3).

Reinforced Concrete Footings.



Elevation

Fig.2

Let a = width of column

,, l = ,, ,, footing

,, V = total shear at base of column

,, v_p = punching unit shear at base of column

,, v = unit shear at any section

,, x = distance to center of gravity of area ABCD

,, b = assumed width of beam to resist bending moment

,, d = depth to center of steel

,, h = total depth of footing

,, A_s = steel area

,, f_s = unit tensile stress in steel

,, Σ_o = sum of perimeters of all bars

,, u = unit bond stress

Reinforced Concrete Footings.

x

C

A

d a l

B

D

Plan

a + 2d

d h

Elevation

Fig.2

Let a = width of column

,, l = ,, ,, footing

,, V = total shear at base of column

,, v_p = punching unit shear at base of column

,, v = unit shear at any section

,, x = distance to center of gravity of area ABCD

,, b = assumed width of beam to resist bending moment

,, d = depth to center of steel

,, h = total depth of footing

,, A_s = steel area

,, f_s = unit tensile stress in steel

,, = sum of perimeters of all bars

,, u = unit bond stress

FORMULAS.

$$V = (l^2 - a^2)w \qquad (4)$$

$$v_p = \frac{V}{4\ a\ 7/8\ d} \qquad (5)$$

$$x = \frac{a + 2l}{a + l}\ \frac{(l - a)1/2}{3} \qquad (6)$$

$$M = \frac{V}{4}\ x\ ,\ (7)\ ,\ A_s = \frac{M}{7/8\ d\ f_s} \qquad (8)$$

$$u = \frac{V}{4\ d\ 7/8\ \Sigma o} \qquad (9)$$

$$b = a + 2\ d + 1/2\ (l - a - 2\ d) \qquad (10)$$

p63

Fig. 2 is shown in order to bring out the conceptions on which the design is based, and also to show more clearly the notation adopted. The formulas above will be discussed in the order shown.

Formula (4) is an expression giving the total shear at the base of the four sides of the column. It is obtained by multiplying the soil pressure by the difference in areas of the footing and the column. The next formula, (5), gives the unit punching shear at the base of the column. This formula is similar to the common formula for shear in a concrete beam, in which "a" and "d" are the width and depth of the beam, and "7/8" is an average value for the effective depth of the beam.

In the design of footings, one-quarter of the load was assumed to be applied on the trapezoid ABCD (see Fig.2). The center of gravity of this load is given in formula (6). This formula is merely the formula for the

center of gravity of any trapezoidal area.Knowing the distance to the center of gravity, and the total load, the bending moment is the product of the two, as given in formula (7). The next formula (8) is the resisting moment for any concrete beam when the steel in any width is known, and the depth and unit stress in the steel are given. Formula (9) gives the unit bond stress for a beam of the same width and depth as above, the other symbols used having the meanings given in the notation. The last formula (10) is a formula giving the assumed width of beam in formulas (8) and (9). This width is that recommended in the above mentioned Bulletin as being a conservative value to use.

In order to obtain comparative designs, the plain and reinforced concrete footings were designed using the same column load and soil pressure for each. Column loads of 100,000, 200,000, and 400,000 lbs, and soil pressures of 3500 and 4500# were adopted. In this way a range of values was obtained which would cover all the ordinary cases occurring in building construction.

TYPICAL COMPUTATIONS.

It is believed the method of design will be elucidated by giving the actual design data. Fig.3 p67 gives the details of plain and reinforced concrete footings, each of which was designed for a column load of 200,000#, and a soil pressure of 3500#. The design of the plain concrete footing will be taken up first. A footing of

the total width and depth shown was assumed. Since the depth of the offset is 15", the width according to formula (2) is .64 x 15 = 9.6". A 10" width was used. On this basis the total width of each course is as given. In order to determine whether or not the soil pressure was exceeded, ~~or not~~, the following computations were made.

Weight of footing	24000
Column load	200000
Total	224000

$w = \frac{224000}{8x8} = 3500\#$. ~~Axslightlyxsmallerxfootingxcould haxexbeenxusedxxbutxnoxchangexwasxmadex~~ This showed that the soil pressure had not been exceeded.

The computations of the reinforced concrete footing weremore elaborate. Assuming the footing given in Fig.3 p.67

Weight of footing	23000
Column load	200000
Total	223000

$w = \frac{223000}{8x8} = 3480\#$. A slightly smaller footing could have been used, but no change was made. According to formula (4), V = (64 - 2) 3500 = 21800#. $v_p = \frac{218000}{4x18x7/8x26} = 133\#$, by formula (5). This shows that the depth is the minimum as determined by punching shear. Referring to Fig. 2, p.63 (a + 2 d) = 18 + 52 = 70" = 5'-10". v on (a + 2 d) = $\frac{(64 - 5.83^2)\ 3500}{4 \times 70 \times 26 \times 7/8} = 17\#$. Since this value for shear on this section is less than one-half the allowable shearing stress for beams reinforced with straight bars only, the use of straight bars without any

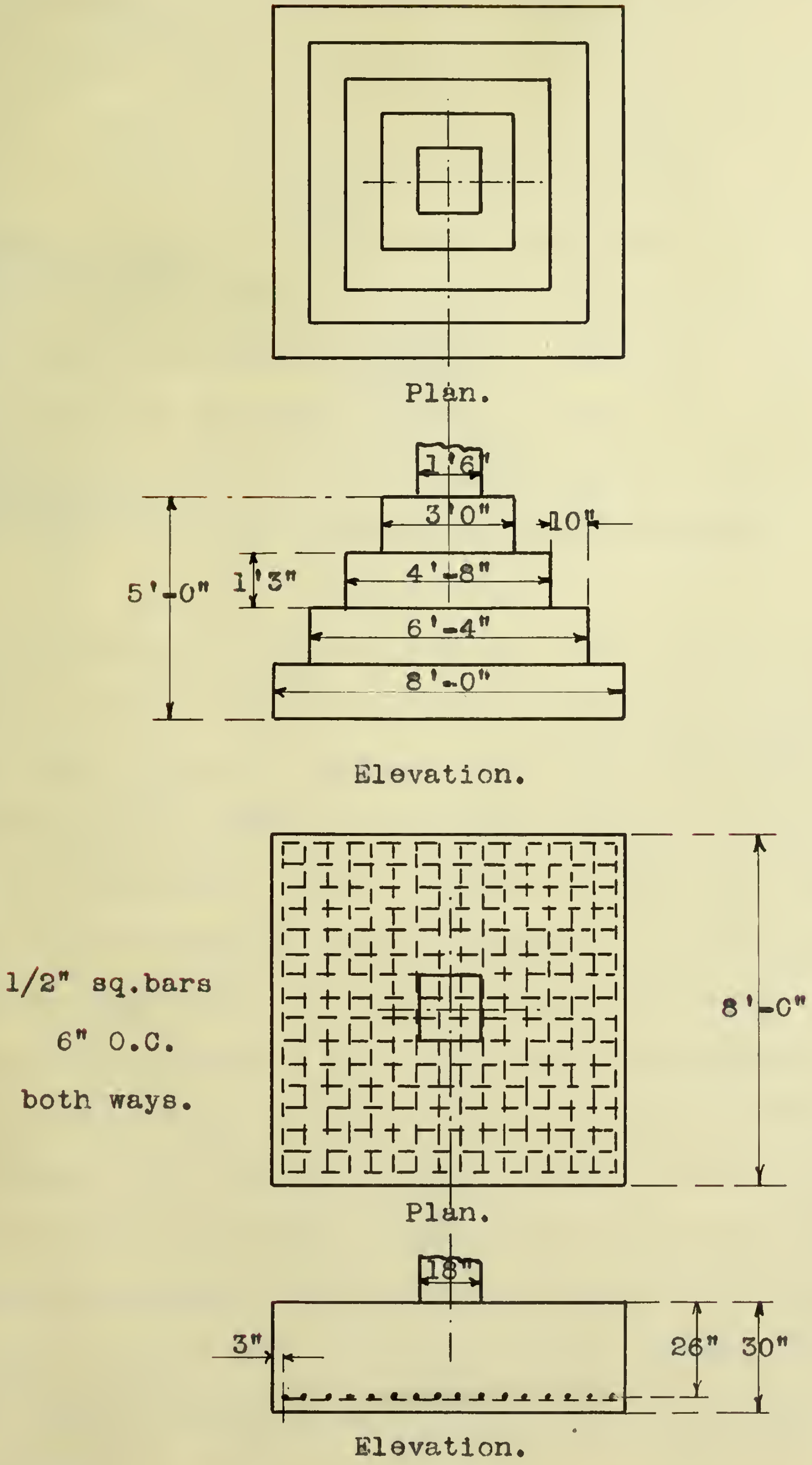

Plan.
1'6"
3'0"
10"
5'-0"
1'3"
4'-8"
6'-4"
8'-0"
Elevation.
1/2" sq.bars
6" O.C.
both ways.
8'-0"
Plan.
18"
3"
26"
30"
Elevation.

Fig. 3.

stirrups will be perfectly safe. $x = \frac{18 + 2x\ 96}{18 + 96} \times \frac{39}{3} =$ 24", according to formula (6). $M = \frac{218000}{4} \times 24 =$ 1,310,000 in. lbs., by formula (7). Formula (10) gives $b = 18 + 2 \times 26 + 1/2\ (96 - 18 - 52) = 83" = 6' - 11"$.

M per foot width $= \frac{1310000}{6.92} = 190{,}000$ in. lbs. $A_s = \frac{190{,}000}{7/8 \times 26 \times 18000} = .47$ sq. ins., according to formula (8). 1/2" square bars, 6" O.C., $A_s = .50$ sq. ins. was used. V per foot width $= \frac{218{,}000}{4 \times 6.92} = 7900\#$. $\Sigma_o = 2 \times 4 \times 1/2 = 4.00$ sq, ins. per foot width. By formula (9) $u = \frac{7900}{7/8 \times 26 \times 4} = 87\#$. From the above analysis it is seen that none of the allowable stresses are exceeded.

UNIT COSTS.

The 1:2:4 concrete in both the plain and the reinforced concrete footings has been estimated at 20¢ per cubic foot. Since the steel is not bent, ~~nor~~ and is not especially difficult ~~of~~ to plac~~ing~~e, it has been estimated at 2- 1/2¢ per lb. Referring to Fig. 3 it will be seen that rough forms will be required for all coures of the plain concrete footing except the bottom course. Since the labor cost of these forms is low, they have been estimated at 5¢ per square foot of ~~form~~ surface in contact with the concrete. No formwork was estimated for the lowest course of the plain concrete footings, or for any of the reinforced concrete footings, as it was assumed that the soil was of such a nature that the hole could be excavated to the exact size of the footing, and that the earth could act as a form.

The excavation for these footings has been estimated at 75¢ per cubic yard. It is hard to give an exact figure for excavation, as the cost depends upon the labor market, ~~as to~~ the difficulties encountered, ~~as to~~ the length of haul to the dumping grounds, etc., etc. It is believed that 75¢ a yard is a fair average in localities remote from the congested business district of a large city.

QUANTITIES AND TOTAL COSTS.

The quantities of the footings shown in Fig. 3 are typical of the whole series, and are given below.

Plain Concrete Footing.

Concrete	168 cu. ft.	at 20¢	=	$ 33.60
Forms	70 sq. ft.	at 5¢	=	3.50
Excav.	11.8 yds.	at 75¢	=	8.86
			Total	$ 45.96

Reinforced Concrete Footing.

Concrete	160 cu. ft.	at 20¢	=	$ 32.00
Steel	206 lbs.	at 2-1/2¢	=	5.15
Excav.	6 yds.	at 75¢	=	4.50
			Total	$ 41.65

CONCLUSION.

It is thought that the unit costs used represent fair averages of the costs in the Middle West. On this basis it was found that the reinforced concrete footings in every case were decidedly more economical than the plain concrete footings designed for the same column load and soil pressure.